Say an alternating series converges absolutely if it is $|C|$ ($\rightarrow C$ by $\square 1$) and converges conditionally if it is $|D|C$.

*1. Which series in the table converge absolutely/conditionally?

*2. The alternating $p$-series is $|C|$ if $p > 1$ and is $|D|C$ if $0 < p \leq 1$.

d. [limit ratio of consecutive terms] Let $\ell := \lim|\frac{a_{n+1}}{a_n}|$

Note: For geometric series we have $\frac{a_{n+1}}{a_n} = r$ (constant), $\therefore \ell = |r|$.

*1. Find $\ell$ for $\sum_{n=1}^{\infty} \frac{\ln n}{n^2}$ and $\sum_{n=1}^{\infty} \frac{n^2}{3^n}$ (see c above), $\sum_{n=1}^{\infty} \frac{(-1)^n}{n^{3/2}}$, $\sum_{n=1}^{\infty} \frac{2^n}{n^3}$, $\sum_{n=1}^{\infty} \frac{\ln^n}{n^n}$, $\sum_{n=1}^{\infty} \frac{n^2 + 10n}{2n^3 + 7}$, $[1, \frac{1}{3}, \frac{2}{3}, 1]$.

By N1 [1.6, *2], the limit ratio for $\sum_{n=1}^{\infty} \frac{P(n)}{Q(n)}$ is 1, where $P, Q$ are polynomials. Thus we have the useful:

* The value of $\ell$ does not change if we drop power functions (e.g. $\sqrt{n}, n^2 - 5n + 3, \frac{n^2 + 10n}{n^3 + 7}$ etc.) in products/quotients.

*2. Use * to find $\ell$ for $\sum_{n=1}^{\infty} \frac{\ln n}{n^{3/2}}$. [1]

e. Ratio test If $\ell < 1$ then the series is $|C|$, if $\ell > 1$ then the series is $|D|D$. [Why? If a series has limit ratio $\ell < 1$ then the series is (for large $n$) smaller than the convergent geometric series $\sum_{n=0}^{\infty} a \ell^n$ where $\ell < \ell < 1$. Similarly if $\ell > 1$ then the series is bigger than the divergent geometric series $\sum_{n=0}^{\infty} a \ell^n$ where $1 < \ell < 1$.]

*3. Use the ratio test to determine if the series $\sum_{n=1}^{\infty} \frac{\ln n}{n^{3/2}}$ converges. [$\ell = \frac{1}{3}, \therefore C$]

Ex 46. 20 – 25, 28, 32, 39, 40

8.5 *1. Find all $x$ for which the series converges. [Hint: Use Ratio test & 8.4.c (above) for ‘end/border points’]

<table>
<thead>
<tr>
<th>$\sum_{n=0}^{\infty} x^n$ (p. 459)</th>
<th>$\frac{1}{n} x^n$</th>
<th>$n^3 x^n$</th>
<th>$\frac{\ln(n) x^n}{n^2}$</th>
<th>$\frac{2^n x^n}{n^3}$</th>
<th>$\frac{n^2 x^n}{n^3}$</th>
<th>$n! x^n$</th>
<th>$\frac{n! x^n}{(n+2)!}$</th>
<th>$\frac{(n+2)! x^n}{n!}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>C if $-1 &lt; x &lt; 1$</td>
<td>$[-1, 1]$</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

a. power series in $(x - a)$

b. $R$(adius) $o(A) C$ 0 $R$ $\infty$

$R$(adius) $o(A)$ C $\{a\}$ $([a - R, a + R]), 4$ cases $(-\infty, \infty)$

Ex. 6, 7, 8, 9, 10, 11, 12, 13, 14

8.8 [cf. N6, 1b.] Ex. 9, 10, 11, 12, 13, 16