8.3 Positive (terms) series. \( \sum_{n=1}^{\infty} a_n \) where \( a_n \geq 0 \). In this case, \( s_n \) is an increasing sequence, \( \sum_{n=1}^{\infty} a_n \) is C if its partial sums remain bounded.

- Left-hand Riemann sum overestimates the integral of a decreasing function. (1a on N6)

b. Direct Comparison Test\(^{45}\),

\[ \sum_{n=1}^{\infty} \frac{1}{n^p} \]

- If \( p < 1 \), \( \frac{1}{n^p} \) diverges.
- If \( p > 1 \), \( \frac{1}{n^p} \) converges.
- If \( p = 1 \), the test is inconclusive.

Example: \( \sum_{n=1}^{\infty} \frac{1}{n} \) diverges (Harmonic Series).

- L'Hôpital's Rule: If \( \lim_{n \to \infty} \frac{a_n}{b_n} = L \), then \( \sum_{n=1}^{\infty} a_n \) and \( \sum_{n=1}^{\infty} b_n \) converge or both diverge.

Ex. 3, 4, 17, 38b, 39b [Hint: L<\(\mathbb{P}\)]

c. Integral test\(^{43}\),

\[ \int_{1}^{\infty} f(x) \, dx \]

Example: \( \sum_{n=1}^{\infty} \frac{1}{n^p} \) is convergent if \( p > 1 \). Otherwise, it diverges.

Ex. 13, 15

d. Limit Comparison Test\(^{44}\), (also see Ex. 38a, 39a)

\[ \lim_{n \to \infty} \frac{a_n}{b_n} \]

- If \( \lim_{n \to \infty} \frac{a_n}{b_n} = L \), where \( L > 0 \) and finite, then \( \sum_{n=1}^{\infty} a_n \) and \( \sum_{n=1}^{\infty} b_n \) both converge or both diverge.

Ex. 14, 22, 26

e. Which series can be found convergent by comparison (direct or limit) with a \( p \)-series? \( \sum_{n=1}^{\infty} \frac{1}{n^p} \)

- Example: \( \sum_{n=1}^{\infty} \frac{1}{n^2} \) is convergent.

Ex. 41, 42

8.4 Non positive series.

- A series \( \sum_{n=1}^{\infty} a_n \) is \textit{absolutely convergent} (write \( \text{C} \)) if the positive series \( \sum_{n=1}^{\infty} |a_n| \) is convergent.

- Example: \( \sum_{n=1}^{\infty} \frac{1}{n^2} \) is \text{C}.

- The converse is false! Counterexample: \( \sum_{n=1}^{\infty} (-1)^{n-1} \frac{1}{n} \) is \text{D}, but \( \sum_{n=1}^{\infty} \frac{1}{n^2} \) is \text{C}.

- Example: \( \sum_{n=1}^{\infty} (-1)^n \frac{1}{n^2} \) is \text{D}.

Ex. 3