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1 Overview of the Distribution

1.1 Discrete Distributions

1.1.1 Binomial Distribution

Notation: $X \sim B(n,p),\ 0 \le p \le 1$

Density: $f(x) = \left(\begin{array}{c} n \\ x \end{array} \right) p^x (1-p)^{n-x},\ x = 0, 1, 2, ..., n$

$E(X) = np,\ V(X) = np(1-p)$

MATLAB fuctions:

random number: *binornd(n,p,#row,#col)*
(ex. r=binornd(100,0.7,20,1))
parameter estimate: *binofit(r,n)* (ex. [phat,pci]=binofit(r,100))
pdf: *binopdf(x,n,p)* (ex. binopdf(1:100,100,0.7))
cdf: *binocdf(x,n,p)* (ex. binocdf(1:100,100,0.7))
inverse cdf: *binoinv(P,n,p)* (ex. binoinv(.95,100,0.7))
practice: Example 2.1, p26

1.1.2 Poisson Distribution

Notation: $X \sim Poisson(\lambda),\ \lambda > 0$

Density: $f(x) = \frac{\lambda^x}{x!} exp(-\lambda),\ x = 0, 1, 2, ...$

$E(X) = \lambda,\ V(X) = \lambda$

MATLAB fuctions:

random number: *poissrnd(λ,#row,#col)*
(ex. r=poissrnd(5,20,1))
parameter estimate: *poissfit(r,α)* (ex. [lamhat,lamci]=poissfit(r,.05))
pdf: *poisspdf(x,λ)* (ex. poisspdf(1:20,5))
cdf: *poisscdf(x,λ)* (ex. poisscdf(1:20,5))
inverse cdf: *poissinv(P,λ)* (ex. poissinv(.95,5))
practice:

```
x=0:20;
pdf=poisspdf(x,5);
cdf=poisscdf(x,5);
subplot(121),bar(x,pdf,1)
title('pdf poiss(5)')
xlabel('X'),ylabel('f(x)')
axis square
subplot(122),bar(x,cdf,1,'w')
title('cdf poiss(5)')
xlabel('X'),ylabel('F(x)')
```

axis square

1.1.3 Some other Discrete Distributions

1. Negative Binomial: $X \sim NegBin(r,p),\ 0 \leq p \leq 1,\ r = 1,2,...$
   MATLAB functions *nbinrnd, nbinfit, nbinpdf, nbincdf*

2. Discrete Uniform:
   MATLAB functions *unidrnd, unidpdf, unidcdf*

3. Geometric: $X \sim Geo(p), 0 \leq p \leq$
   MATLAB functions *geornd, geopdf, geocdf*

4. Hypergeometric: $X \sim Hyper(M,K,n)$
   MATLAB functions *hygernd, hygepdf, hygecdf*

1.2 Continuous Distributions

1.2.1 Normal Distribution

Notation: $X \sim N(\mu,\sigma^2),\ \sigma > 0$

Density: $f(x) = (2\pi\sigma^2)^{-\frac{1}{2}} exp\left\{-\frac{1}{2\sigma^2}(x-\mu)^2\right\}$

$E(X) = \mu,\ V(X) = \sigma^2$

MATLAB fuctions:

random number: *normrnd(μ,σ,#row,#col)* (ex. x=normrnd(0,1,20,1))
parameter estimate: *normfit(x)* (ex. [mu,s,muci,sci]=normfit(x))
pdf: *normpdf(x,μ,σ)* (ex. normpdf(-3:.1:3,0,1))
cdf: *normcdf(x,μ,σ)* (ex. normcdf(-3:.1:3,0,1))
inverse cdf: *norminv(P,μ,σ)* (ex. norminv(0.95,0,1))
practice: example 2.5, p32

```
x=-3:.1:3;
pdf=normpdf(x,0,1);
cdf=normcdf(x,0,1);
subplot(121),plot(x,pdf,'-')
title('pdf N(0,1)')
xlabel('X'),ylabel('f(x)')
axis([-3.5 3.5 0 0.5])
subplot(122),plot(x,cdf,'-')
title('cdf N(0,1)')
xlabel('X'),ylabel('F(x)')
axis([-3.5 3.5 0 1])
```

1.2.2 Student's t Distribution

Notation: $X \sim t_\nu,\ \nu > 0$

Density: $f(x) = \frac{\Gamma((\nu+1)/2)}{\Gamma(\nu/2)\sqrt{\pi\nu}}(1 + x^2/\nu)^{-(\nu+1)/2}$

$E(X) = 0$ if $\nu > 1$, $\quad V(X) = \frac{\nu}{\nu+2}$ if $\nu > 2$

MATLAB fuctions:

random number: *trnd(ν,#row,#col)* (ex. x=trnd(7,20,1))

pdf: *tpdf(x,ν)* (ex. tpdf(-5:.1:5,7))

cdf: *tcdf(x,ν)* (ex. tcdf(-5:.1:5,7))

inverse cdf: *tinv(P,ν)* (ex. tinv(.95,7))

practice:

```
x=-5:.1:5;
y1=normpdf(x,0,1);
y2=tpdf(x,7);

subplot(121),plot(x,y1,'-',x,y2,'-.')
title('pdf of N(0,1) vs. t(7)')
xlabel('X'),ylabel('f(x)')
axis square
legend('normal','t',0)

z1=normcdf(x,0,1);
z2=tcdf(x,7);

subplot(122),plot(x,z1,'-',x,z2,'-.')
title('cdf N(0,1) vs. t(7)')
xlabel('X'),ylabel('F(x)')
axis square
legend('normal','t',0)
```

1.2.3 Chi-Square Distribution

Notation: $X \sim \chi^2_\nu,\ \nu > 0$

Density: $f(x) = \frac{x^{(\nu-2)/2}}{2^{\nu/2}\Gamma(\nu/2)} exp(-x/2)$

$E(X) = \nu\ ,\quad V(X) = 2\nu$

MATLAB fuctions:

random number: *chi2rnd(ν,#row,#col)* (ex. x=chi2rnd(7,20,1))

pdf: *chi2pdf(x,ν)* (ex. chi2pdf(0:.1:20,7))

cdf: *chi2cdf(x,ν)* (ex. chi2cdf(0:.1:20,7))

inverse cdf: *chi2inv(P,ν)* (ex. chi2inv(.95,7))

practice: plot the pdfs of the Chi-squre distribution at various dfs and compare.

1.2.4 F Distribution

Notation: $X \sim F_{\nu_1,\nu_2},\ \nu_1 > 0,\ \nu_2 > 0$

Density: $f(x) = \frac{\Gamma((\nu_1+\nu_2)/2)}{\Gamma(\nu_1/2)\Gamma(\nu_2/2)} \frac{(\nu_1/\nu_2)^{\nu_1/2} x^{(\nu_1-2)/2}}{\left[1+\left(\frac{\nu_1}{\nu_2}\right)x\right]^{(\nu_1+\nu_2)/2}}$

$E(X) = \frac{\nu_2}{\nu_2-2}$ if $\nu_2 > 2$, $\quad V(X) = \frac{2\nu_2^2(\nu_1+\nu_2-2)}{\nu_1(\nu_2-2)^2(\nu_2-4)}$ if $\nu_2 > 4$

MATLAB fuctions:

random number: *frnd(ν_1, ν_2,#row,#col)*
(ex. x=frnd(3,7,20,1))

pdf: *fpdf(x,ν_1, ν_2)* (ex. fpdf(0:.1:5,3,7))

cdf: *fcdf(x,ν_1, ν_2)* (ex. fcdf(0:.1:5,3,7))

inverse cdf: *finv(P,ν_1, ν_2)* (ex. finv(.95,3,7))

practice:

```
x=0:.1:5;
y1=fpdf(x,1,1);
y2=fpdf(x,1,10);
y3=fpdf(x,10,1);
y4=fpdf(x,10,10);

subplot(221),plot(x,y1)
title('pdf of F(1,1)')
xlabel('X'),ylabel('f(x)')
axis square
subplot(222),plot(x,y2)
title('pdf of F(1,10)')
xlabel('X'),ylabel('f(x)')
axis square
subplot(223),plot(x,y3)
title('pdf of F(10,1)')
xlabel('X'),ylabel('f(x)')
axis square
subplot(224),plot(x,y4)
title('pdf of F(10,10)')
xlabel('X'),ylabel('f(x)')
axis square
```

1.2.5 Some other Continuous Distributions

1. Beta: $X \sim Beta(\alpha, \beta),\ \alpha > 0,\ \beta > 0$
   MATLAB functions *betarnd, betafit, betapdf, betacdf, betainv*

2. Exponential: $X \sim Exp(\lambda),\ \lambda > 0$
   MATLAB functions *exprnd, expfit, exppdf, expcdf, expinv*

3. Gamma: $X \sim Gamma(r, \lambda),\ r > 0,\ \lambda > 0$
   MATLAB functions *gamrnd, gamfit, gampdf, gamcdf, gaminv*

4. Lognormal: $X \sim Lognormal(\mu, \sigma^2),\ \sigma > 0$
   MATLAB functions *lognrnd, longnfit, lognpdf, logncdf, longninv*

5. Uniform: $X \sim Unif(a, b),\ a < b$
   MATLAB functions *unifrnd, unifpdf, unifcdf, unifinv*

6. Weibull: $X \sim Weibull(a, b),\ a > 0,\ b > 0$
   MATLAB functions *weibrnd, wblfit, weibpdf, weibcdf, weibinv*