

1. The average scores and covariance matrix are obtained from n=92 high school students on a college entrance exam with two subtests ( $X_1$  and  $X_2$ ).

$$\bar{x} = \begin{bmatrix} 65 \\ 78 \end{bmatrix}, \quad S = \begin{bmatrix} 12 & 5 \\ & 8 \end{bmatrix}$$

- (a) Test the hypothesis  $\mu' = [70, 80]$  at 0.05 level.
- (b) Test for the equality of mean scores at 0.05 level.
- (c) Compute 95% simultaneous CI for  $\mu_1$  and  $\mu_2$ . Also, compute 95% simultaneous CI for  $\mu_1 - \mu_2$ . Does this CI support your answer in (a)?

2. Observations on two responses are collected for three groups. The observation vectors are

Group 1:  $\begin{bmatrix} 6 \\ 7 \end{bmatrix}, \begin{bmatrix} 5 \\ 9 \end{bmatrix}, \begin{bmatrix} 8 \\ 6 \end{bmatrix}, \begin{bmatrix} 4 \\ 9 \end{bmatrix}, \begin{bmatrix} 7 \\ 9 \end{bmatrix}$ ,

Group 2:  $\begin{bmatrix} 3 \\ 3 \end{bmatrix}, \begin{bmatrix} 1 \\ 6 \end{bmatrix}, \begin{bmatrix} 2 \\ 3 \end{bmatrix}$ ,

Group 3:  $\begin{bmatrix} 2 \\ 3 \end{bmatrix}, \begin{bmatrix} 5 \\ 1 \end{bmatrix}, \begin{bmatrix} 3 \\ 1 \end{bmatrix}, \begin{bmatrix} 2 \\ 3 \end{bmatrix}$

It is calculated that

$$\bar{x} = \begin{bmatrix} 4 \\ 5 \end{bmatrix}, \quad \bar{x}_1 = \begin{bmatrix} 6 \\ 8 \end{bmatrix}, \quad \bar{x}_2 = \begin{bmatrix} 2 \\ 4 \end{bmatrix}, \quad \bar{x}_3 = \begin{bmatrix} 3 \\ 2 \end{bmatrix} \text{ and}$$

$$S = \frac{1}{11} \begin{bmatrix} 54 & 35 \\ 35 & 102 \end{bmatrix}, \quad S_1 = \begin{bmatrix} 2.5 & -1.5 \\ -1.5 & 2 \end{bmatrix}, \quad S_2 = \begin{bmatrix} 1 & -1.5 \\ -1.5 & 3 \end{bmatrix}, \quad S_3 = \begin{bmatrix} 2 & -1.33 \\ -1.33 & 1.33 \end{bmatrix}$$

- (a) Using the summary statistics complete the MANOVA table below.

Source	Degrees of freedom	SSCP
Group		$\begin{bmatrix} & \\ & \end{bmatrix}$
Error		$\begin{bmatrix} & \\ & \end{bmatrix}$
Total		$\begin{bmatrix} & \\ & \end{bmatrix}$

- (b) Evaluate Wilks' lambda and perform hypothesis test for group effect at 0.05 level. Repeat the test using the Bartlett's chi-square approximation.
- (c) Construct 95% simultaneous CI for the difference between Group1 and Group2 for each response.
- (d) Now, consider Group1 and Group2 only. Sketch the 95% confidence ellipse for  $\mu_1 - \mu_2$ . Determine the lengths and directions for the axes of the ellipse.