#1. Method

<table>
<thead>
<tr>
<th>Blend</th>
<th>A</th>
<th>B</th>
<th>C</th>
<th>D</th>
<th>$Y_{ij}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>89</td>
<td>88</td>
<td>97</td>
<td>94</td>
<td>368</td>
</tr>
<tr>
<td>2</td>
<td>84</td>
<td>77</td>
<td>88</td>
<td>79</td>
<td>328</td>
</tr>
<tr>
<td>3</td>
<td>81</td>
<td>87</td>
<td>97</td>
<td>85</td>
<td>350</td>
</tr>
<tr>
<td>$Y_{i.}$</td>
<td>254</td>
<td>252</td>
<td>282</td>
<td>258</td>
<td>$Y_{..} = 1046$</td>
</tr>
</tbody>
</table>

(a) RCBBD

$Y_{ij} = \mu + \tau_i + \beta_j + \epsilon_{ij}$, $i = 1, 2, 3, 4$

$\tau_i$ : Method effect

$\beta_j$ : Blend (block) effect

Assumptions:

$\sum \tau_i = \sum \beta_j = 0$

$\epsilon_{ij} \sim \text{N}(0, \sigma^2)$

Estimates

$\hat{\mu} = \bar{Y}_{..} = \frac{1046}{12} = 87.17$

$\hat{\tau}_1 = \bar{Y}_{1.} - \bar{Y}_{..} = \frac{254}{3} - 87.17 = -2.503$

$\hat{\tau}_2 = \bar{Y}_{2.} - \bar{Y}_{..} = \frac{252}{3} - 87.17 = -3.17$

$\hat{\tau}_3 = \bar{Y}_{3.} - \bar{Y}_{..} = \frac{282}{3} - 87.17 = 6.83$

$\hat{\tau}_4 = \bar{Y}_{4.} - \bar{Y}_{..} = \frac{258}{3} - 87.17 = 1.17$

$\hat{\beta}_1 = \bar{Y}_{.1} - \bar{Y}_{..} = \frac{368}{4} - 87.17 = 4.83$

$\hat{\beta}_2 = \bar{Y}_{.2} - \bar{Y}_{..} = \frac{328}{4} - 87.17 = -5.17$

$\hat{\beta}_3 = \bar{Y}_{.3} - \bar{Y}_{..} = \frac{350}{4} - 87.17 = 0.33$

$\hat{\sigma}^2 = \text{MSE} = 12.3$
(b) 
\[ SS_T = \sum_{i} \sum_{j} Y_{ij}^2 - \frac{Y_{..}^2}{N} = (89^2 + 88^2 + \ldots + 85^2) - \frac{1046^2}{12} = 467.7 \]
\[ SS_{Trt} = \frac{1}{3} \sum_{i} Y_{i.}^2 - \frac{Y_{..}^2}{N} = \frac{1}{3} \left( 254^2 + 252^2 + 282^2 + 258^2 \right) - \frac{1046^2}{12} = 193 \]
\[ SS_{BL} = \frac{1}{4} \sum_{j} Y_{.j}^2 - \frac{Y_{..}^2}{N} = \frac{1}{4} \left( 368^2 + 328^2 + 350^2 \right) - \frac{1046^2}{12} = 200.7 \]
\[ SS_E = SS_T - SS_{Trt} - SS_{BL} = 467.7 - 193 - 200.7 = 74 \]

<table>
<thead>
<tr>
<th>Source</th>
<th>SS</th>
<th>df</th>
<th>MS</th>
<th>F</th>
<th>P-value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Method</td>
<td>193</td>
<td>3</td>
<td>64.3</td>
<td>5.23</td>
<td>&lt; 0.05</td>
</tr>
<tr>
<td>Block (block)</td>
<td>200.7</td>
<td>2</td>
<td>100.35</td>
<td>8.16</td>
<td>&lt; 0.05</td>
</tr>
<tr>
<td>Error</td>
<td>74</td>
<td>6</td>
<td>12.3</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Total</td>
<td>467.7</td>
<td>11</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Test for Method effect

\[ H_0 : \alpha_i = 0 \quad \forall \ i = 1, 2, 3, 4 \]
\[ H_1 : \text{at least one } \alpha_i \text{ is not zero} \]

\[ F^* = \frac{MS_{Trt}}{MS_E} = \frac{64.3}{12.3} = 5.23 \]

\[ F_{.05} (3, 6) = 4.76 \]

Since \( F^* > 4.76 \) (or \( p\text{-value} < 0.05 \)), we reject \( H_0 \) at 0.05 level, and conclude that there exists significant difference between the methods.

Test for Block (block) effect

\[ H_0 : \beta_j = 0 \quad \forall \ j = 1, 2, 3 \]
\[ H_1 : \text{at least one } \beta_j \text{ is not zero} \]

\[ F^* = \frac{MS_{BL}}{MS_E} = \frac{100.35}{12.3} = 8.16 \], and \[ F_{.05} (2, 6) = 5.14 \]

Since \( F^* > 5.14 \) (\( p\text{-value} < .05 \)), reject \( H_0 \). There exists significant block effect.
(c) Tukey MSD

\[ q_{0.05} (4, 6) = 4.9 \]

\[ \text{MSD} = q_{0.05} \sqrt{\frac{\text{MSE}}{n}} = 4.9 \sqrt{\frac{12.3}{3}} \approx 9.92 \]

A vs B : \[ |\bar{Y}_1 - \bar{Y}_2| = |84.7 - 84| = 0.7 < \text{MSD} \]
A vs C : \[ |\bar{Y}_1 - \bar{Y}_3| = |84.7 - 94| = 9.3 < \text{MSD} \]
A vs D : \[ |\bar{Y}_1 - \bar{Y}_4| = |84.7 - 86| = 1.3 < \text{MSD} \]
B vs C : \[ |\bar{Y}_2 - \bar{Y}_3| = |84 - 94| = 10 > \text{MSD \*} \]
B vs D : \[ |\bar{Y}_2 - \bar{Y}_4| = |84 - 86| = 2 < \text{MSD} \]
C vs D : \[ |\bar{Y}_3 - \bar{Y}_4| = |94 - 86| = 8 < \text{MSD} \]

Tukey procedure indicates that there exist significant mean difference between method B and C.

(d) ANOVA without Blend effect.

\[ \text{SS}_E = 200.7 + 74 = 274.7 \]

<table>
<thead>
<tr>
<th>Source</th>
<th>SS</th>
<th>df</th>
<th>MS</th>
<th>F</th>
<th>P-val</th>
</tr>
</thead>
<tbody>
<tr>
<td>Method</td>
<td>193</td>
<td>3</td>
<td>64.3</td>
<td>1.87</td>
<td>&gt; 0.10</td>
</tr>
<tr>
<td>Error</td>
<td>274.7</td>
<td>8</td>
<td>34.3</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Total</td>
<td>467.7</td>
<td>11</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Now \[ F = \frac{\text{MS}_{\text{tr}+}}{\text{MS}_e} = \frac{64.3}{34.3} = 1.87 \]

\[ F_{0.05} (3, 8) = 4.07 \]

Since \[ F < 4.07 \] (or p-value > 0.05), we fail to reject \( H_0 \) and conclude the method effect is NOT significant which against the result with block. Hence blocking is necessary.
(a) One-way ANOVA

\[ Y_{ij} = \mu + \tau_i + \epsilon_{ij}, \quad i = 1, 2, 3 \]
\[ j = 1, \ldots, 5 \]

\[ \sum_i \tau_i = 0 \]
\[ \epsilon_{ij} \sim i.d.d \ N(0, \sigma^2) \]

Estimates

\[ \hat{\mu} = \frac{\sum Y_{ij}}{15} = \frac{216}{15} = 14.4 \]
\[ \hat{\tau}_1 = \bar{Y}_1. - \bar{Y}.. = \frac{90}{5} - 14.4 = 3.6 \]
\[ \hat{\tau}_2 = \bar{Y}_2. - \bar{Y}.. = \frac{72}{5} - 14.4 = 0 \]
\[ \hat{\tau}_3 = \bar{Y}_3. - \bar{Y}.. = \frac{54}{5} - 14.4 = -3.6 \]
\[ \hat{\sigma}^2 = MSE = 16.7 \]

(b)

<table>
<thead>
<tr>
<th>Source</th>
<th>df</th>
<th>SS</th>
<th>MS</th>
<th>F</th>
<th>P-val</th>
</tr>
</thead>
<tbody>
<tr>
<td>Model</td>
<td>2</td>
<td>129.6</td>
<td>64.8</td>
<td>3.89</td>
<td>&lt;.05</td>
</tr>
<tr>
<td>Error</td>
<td>12</td>
<td>200</td>
<td>16.67</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Total</td>
<td>14</td>
<td>329.6</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Test for Drying method

\[ H_0: \tau_i = 0 \quad \forall i = 1, 2, 3 \]
\[ H_1: \text{at least one } \tau \text{ is not zero} \]

\[ F^* = \frac{64.8}{16.7} = 3.89, \quad F_{0.05}(2, 12) = 3.89 \]

Here \( F^* = 3.89 \) is at the critical value (p-val=.05), so we would barely reject \( H_0 \) and conclude that the method effect is significant at .05 level.
There exists possible violation to the equal variance assumption.

\[ \bar{Y}_1 = 18, \quad \bar{Y}_2 = 14.4, \quad \bar{Y}_3 = 10.8 \]

\[ S_1 = 4.24, \quad S_2 = 5.59, \quad S_3 = 0.84 \]

<table>
<thead>
<tr>
<th>Level</th>
<th>1</th>
<th>( \frac{\bar{Y}}{S^2} )</th>
<th>2</th>
<th>( \frac{\bar{Y}}{S} )</th>
<th>3</th>
<th>( \frac{\bar{Y}^2}{S} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>1.0</td>
<td>4.2</td>
<td>76.4</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>B</td>
<td>0.46</td>
<td>2.58</td>
<td>37.1</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>C</td>
<td>15.3</td>
<td>12.9</td>
<td>138.9</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

The ratio \( \frac{\bar{Y}}{S} \) is the most stable, so we can suggest log transformation.

\[(d)\]

\[ \mu = (15 + 5 + 15) / 3 = 15 \]

\[ \tau_1 = 25 - 15 = 10, \quad \tau_2 = 15 - 15 = 0, \quad \tau_3 = 5 - 15 = -10 \]

\[ \Phi^2 = \frac{n (10^2 + 0^2 + (-10)^2)}{3 (5)^2} = 2.7n \]

\[ a = 3 \]

\[ V_1 = 3 - 1 = 2 \]

\[
\begin{array}{cccccc}
\Phi^2 & \Phi & a(n-1) & \beta & \text{Power} \\
3 & 8.1 & 2.84 & 6 & 0.08 & 0.92 & \checkmark \\
\end{array}
\]

\[ n = 3 \] is enough.
# 3 (a) BIBD

\[ a = b = 3 \]
\[ r = k = 2 \]
\[ n = r(k-1) \]
\[ a - 1 = \frac{2(2-1)}{2} = 1 \] \checkmark

Model: \( Y_{i:j} = \mu + \tau_i + \beta_j + \epsilon_{i:j}, \quad i = 1, 2, 3 \)
\( \epsilon_{i:j} \sim \mathcal{N}(0, \sigma^2) \)

Since \( n = 1 \), it is a valid BIBD and is "minimal".

(b) \[ SS_T = \left( 7^2 + 9^2 + 11^2 + 12^2 + 8^2 \right) - \frac{53^2}{6} = 26.83 \]
\[ SS_{BL} = \frac{1}{2} \left( 13^2 + 20^2 + 20^2 \right) - \frac{53^2}{6} = 16.33 \]
\[ Q_1 = 16 - \frac{1}{2} (13+20) = -0.5 \]
\[ Q_2 = 23 - \frac{1}{2} (20+20) = 3 \]
\[ Q_3 = 14 - \frac{1}{2} (13+20) = -2.5 \]
\[ SS_{Trt (adj)} = \frac{2 \left( (-0.5)^2 + 3^2 + (-2.5)^2 \right)}{(1)(3)} = 10.33 \]
\[ SS_E = 26.83 - 16.33 - 10.33 = 0.17 \]

<table>
<thead>
<tr>
<th>Source</th>
<th>SS</th>
<th>df</th>
<th>MS</th>
<th>F</th>
<th>p-val</th>
</tr>
</thead>
<tbody>
<tr>
<td>Treatment</td>
<td>10.33</td>
<td>2</td>
<td>5.165</td>
<td>30.4</td>
<td>&lt; 0.01</td>
</tr>
<tr>
<td>(adj)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Block</td>
<td>16.33</td>
<td>2</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Error</td>
<td>0.17</td>
<td>1</td>
<td>0.17</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Total</td>
<td>26.83</td>
<td>5</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

\( F^* = \frac{5.165}{0.17} = 30.4 \), \( F_{10}(2,1) = 49.5 \)

Since \( p-value > 0.10 \), we do not reject \( H_0: \tau_i = 0 \) and conclude that the treatment effect is not significant.