1. Suppose that 
\[ x_t = \beta_0 + \beta_1 t + \beta_2 t^2 + w_t, \]
where \( \beta_0, \beta_1, \) and \( \beta_2 \) are constant and \( w_t \) is white noise with mean zero and variance \( \sigma_w^2. \)

(a) Show that \( x_t \) is nonstationary series
(b) Show that the second difference \( \Delta^2 x_t \) is a stationary series.

2. Suppose that 
\[ x_t = \sum_{j=0}^{m} w_{t-j}/(m + 1). \]

(a) Show that the series \( x_t \) is stationary.
(b) Give the ACF of this process. Briefly discuss the ACF.
(c) Give the power spectrum of the process.

3. Consider two time series
\[ x_t = \frac{1}{2}(w_t - w_{t-1}) \]
\[ y_t = \frac{1}{2}(w_t + w_{t-1}) \]
where the white noise series \( w_t \) has mean zero and variance one.

(a) Show that \( x_t \) and \( y_t \) are jointly stationary. Note in addition to autocovariances the cross covariance function must also be a function only of the lag \( h. \)
(b) Express power spectra \( f_x(\nu) \) and \( f_y(\nu) \) in terms of \( \cos(2\pi \nu). \)
(c) Express cross spectrum \( f_{yx}(\nu) \) and squared coherence \( \rho^2_{yx}(\nu) \) in terms of \( \cos(2\pi \nu). \)

Spectral representation
\[ R_x(h) = \int_{-1/2}^{1/2} f_x(\nu)e^{2\pi i\nu h} d\nu \]
and
\[ f_x(\nu) = \sum_{h=-\infty}^{\infty} R_x(h)e^{-2\pi i\nu h} \]

Trignometric identities
\[ e^{ix} = \cos(x) + i\sin(x) \]
\[ e^{-ix} = \cos(x) - i\sin(x) \]
\[ \cos(x) = (e^{ix} + e^{-ix})/2 \]
\[ \sin(x) = (e^{ix} - e^{-ix})/2 \]