PREPARE FOR THE FINAL EXAM

Here are some guidelines that you can use to prepare for the final exam.

- The final exam is cumulative. Some topics will not be on the final exam; these are listed at the end of this guide.
- More than half of the final exam is on Chapter 13. Most of the problems on the exam combine facts we know about curves, surfaces, and solids from Chapters 12 and 13.
- Make sure you know how to do all the problems on midterm exams 2 and 3 and on the quizzes for Chapters 12 and 13. All the problems on the Final Exam are related to problems on the midterm tests or the quizzes.

1. You need to know about vector fields in the plane \( \mathbf{F}(x, y) \) or in space \( \mathbf{F}(x, y, z) \). If you are given vector field \( \mathbf{F} \), you should be able to:
   a. Sketch some vectors of the field \( \mathbf{F} \).
   b. Find \( \text{curl} \mathbf{F} \) and \( \text{div} \mathbf{F} \).
   c. Determine if the field is conservative, and if so, find a potential \( f \) (only for fields in the plane.)

2. You need to know about curves in space. If you are given parametric equations (or a vector equation) for a curve \( C \) in space, you should be able to:
   a. Sketch the curve.
   b. Find the length of the curve: \( \int_C ds \).
   c. Find the work done in moving along the curve in a field \( \mathbf{F} \): \( \text{Work} = \int_C \mathbf{F} \cdot d\mathbf{s} = \int_C \mathbf{F} \cdot d\mathbf{r} \).
   d. If the field \( \mathbf{F} \) in part (c) is conservative, then we can calculate the work in part (c) by
      - Using a simpler curve \( C_1 \): \( \int_{C_1} \mathbf{F} \cdot d\mathbf{r} = \int_{C_1} \mathbf{F} \cdot d\mathbf{r} \).
      - Using the potential \( f \) (in 1(b)): \( \int_C \mathbf{F} \cdot d\mathbf{r} = f(a_2, b_2) - f(a_1, b_1) \), where \( C \) starts at \( (a_1, b_1) \) and ends at \( (a_2, b_2) \).

3. You need to know about surfaces in space. If you are given a surface \( W \) in space represented as the graph of a function \( z = f(x, y) \), you should be able to:
   a. Sketch the surface.
   b. Find the normal to the surface (Using the function we called \( G(x, y, z) \)).
   c. Find the tangent plane to the surface at a given point.
   d. Find the area of the surface: \( \int_W dS \) where \( dS = |\nabla G| dA \).
   e. Find the flux of a field \( \mathbf{F} \) across the surface: \( \int_W \mathbf{F} \cdot \mathbf{n} dS \).

4. You need to know about solid regions in space. If you are given a solid region \( Q \), represented as a region bounded by several surfaces, you should be able to:
   a. Sketch the solid.
   b. Find the volume of the solid: \( \iiint_Q dV \).
   c. Find the mass of the solid if we know the density: \( \iiint_Q d((x, y, z))dV \).
5. You need to know how to use the Divergence Theorem. If you are given a solid $Q$ whose boundary is a closed surface $W$ you should be able to:
   a. Find the flux of a field $\mathbf{F}$ directly out of $Q$ by using the Divergence Theorem:
      \[
      \text{Flux} = \iiint_{Q} \text{div} \mathbf{F} \, dV = \iint_{W} \mathbf{F} \cdot d\mathbf{S}
      \]

6. You need to know how to use Stokes' Theorem. If you are given a surface $W$ (not a closed surface) whose boundary is a closed curve $C$, you should be able to:
   a. Find the work done in moving along $C$ in a vector field $\mathbf{F}$ by using Stokes’ Theorem:
      \[
      \text{work} = \oint_{C} \mathbf{F} \cdot d\mathbf{r} = \iint_{W} \text{curl} \mathbf{F} \cdot d\mathbf{S}
      \]
   b. Evaluate $\iint_{W} \text{curl} \mathbf{F} \cdot d\mathbf{S}$ by using a simpler surface $W_1$ that also has $C$ as its boundary:
      \[
      \iint_{W} \text{curl} \mathbf{F} \cdot d\mathbf{S} = \iint_{W_1} \text{curl} \mathbf{F} \cdot d\mathbf{S}
      \]

Topics NOT on this exam:
- Maxima and minima for functions of several variables
- Lagrange multipliers
- $T$, $N$ and $B$ for curves in space
- Chain rule for functions of several variables
- Directional derivatives