Overview of Session

What is hypothesis testing?
- Hypothesis tests concerning population means
  - Standard Approaches
  - Bootstrapping

What Is A Hypothesis?
- A hypothesis is a statement
  - About a population parameter
  - Which is supported or refuted with sample data
- E.g., People who smoke have
  - higher rates of lung cancer
  - than non-smokers.
- The statement is either true or false.
## Logic of Hypothesis Testing

- Hypothesis testing involves
  - Supporting an interesting idea by
  - Showing its opposite is unlikely to be true.
- Null hypothesis is
  - Symbolized as $H_0$
  - When $H_0$ is rejected
    - Results are "statistically significant"
- Alternative hypothesis is $H_1$
  - Supported when $H_0$ rejected
  - Also called research hypothesis

## Hypothesis Testing Is a Decision Made by the Researcher

### Real World

<table>
<thead>
<tr>
<th>Decision</th>
<th>No Effect</th>
<th>Effect</th>
</tr>
</thead>
<tbody>
<tr>
<td>Reject $H_0$</td>
<td>Type I Error $\alpha$</td>
<td>Correct $1-\beta$</td>
</tr>
<tr>
<td>Fail to Reject $H_0$</td>
<td>Correct $1-\alpha$</td>
<td>Type II Error $\beta$</td>
</tr>
</tbody>
</table>

### What Influences Errors In Decisions?

- $\alpha$ is determined by researcher
  - Usually .05 (5 errors in 100)
- $\beta$ depends on:
  - Sample size
  - Test statistics selected
  - Actual difference from null hypothesis (Effect Size)
  - Not mentioned in text
    - Measurement Error
    - Procedural Error
Hypothesis Tests Involving Means

- Single mean when $\sigma$ is known
- Single mean when $\sigma$ is unknown
- Two related samples
- Two independent samples

Steps In Hypothesis Testing

Single Mean: $\sigma$ is known

1. State Hypotheses: $H_0: \mu = \mu_0$ and $H_1: \mu \neq \mu_0$
2. State $\alpha$ level
3. Determine critical region.
4. Compute $Z = \frac{\bar{X} - \mu}{\sigma / \sqrt{n}}$
5. “Reject” or “Fail to Reject” $H_0$

Single Mean: $\sigma$ is unknown

1. State Hypotheses: $H_0: \mu = \mu_0$ and $H_1: \mu \neq \mu_0$
2. State $\alpha$ level
3. Determine critical region.
4. Compute $t = \frac{\bar{X} - \mu}{s / \sqrt{n}}$
5. “Reject” or “Fail to Reject” $H_0$
Steps In Hypothesis Testing
For Two Related Samples

1. State Hypotheses: $H_0: \mu_D = 0$ and $H_1: \mu_D \neq 0$
2. State $\alpha$ level
3. Determine critical region.
4. Compute $t = \frac{\bar{D} - 0}{s_D / \sqrt{n}}$
5. “Reject” or “Fail to Reject” $H_0$

Steps In Hypothesis Testing
For Two Independent Samples

1. State Hypotheses: $H_0: \mu_1 = \mu_2$ and $H_1: \mu_1 \neq \mu_2$
2. State $\alpha$ level
3. Determine critical region.
4. Compute $t = \frac{(\bar{X}_1 - \bar{X}_2) - (\mu_1 - \mu_2)}{s_p \sqrt{\frac{1}{n_1} + \frac{1}{n_2}}}$
5. “Reject” or “Fail to Reject” $H_0$

Each of the Four Tests Can Be Directional

Non-directional hypotheses
- $H_0: \mu = \mu_0$ and $H_1: \mu \neq \mu_0$ (Z or t)
- $H_0: \mu_1 = \mu_2$ and $H_1: \mu_1 \neq \mu_2$
- $H_0: p_1 = p_2$ and $H_1: p_1 \neq p_2$
- $H_0: p_1 = p_2$ and $H_1: p_1 \neq p_2$
Each of the Four Tests Can Be Directional

Directional (Decreased)
$H_o : \mu \geq \mu_0$ and $H_1 : \mu < \mu_0$ (Z or t)
$H_o : \mu_0 \geq 0$ and $H_1 : \mu_0 < 0$
$H_o : \mu \geq \mu_0$ and $H_1 : \mu < \mu_0$

Directional (Increased)
$H_o : \mu \leq \mu_0$ and $H_1 : \mu > \mu_0$ (Z or t)
$H_o : \mu_0 \leq 0$ and $H_1 : \mu_0 > 0$
$H_o : \mu \leq \mu_0$ and $H_1 : \mu > \mu_0$

Directional Hypotheses: Tests Have Greater Statistical Power

Non-directional hypotheses use two tailed tests

Directional hypotheses use one tailed tests

Relationship Between Hypothesis Tests and Confidence Intervals

- Non-directional hypotheses (2-tailed tests)
  - One-to-one correspondence
  - Each hypothesis involves a point estimate of an interval
  - If interval does not include $H_0$ then significant
  - E.g., 95% interval $[0.1, 25.22]$ is significant at .05 when $H_0 : \mu = 0$ because 0 is not in interval.

- Directional hypotheses (1-tailed tests)
  - Same idea
  - When interval level is $1 - 2\alpha$
  - E.g., For $\alpha = .05$, interval $\approx 90\%$.
Bootstrap Hypothesis Tests

- The relationship between
  - Confidence intervals & hypotheses tests
  - Used to conduct test
- Steps:
  - Define the appropriate confidence level for test
  - Construct interval for that level
  - Determine if value for $H_0$ is in the interval
  - Reject $H_0$ if value not in interval.