Consider the veracity or falsehood of each of the following statements. For bonus, argue for those that you believe are true while providing a counterexample for those that you believe are false.

1. If \( \mathbf{A}^4 - \mathbf{I} = \mathbf{A} \), then \( \mathbf{A} \) is invertible.
2. If \( \mathbf{A} = \begin{pmatrix} 0 & a \\ 1 & b \end{pmatrix} \), then its centralizer is \( \langle \mathbf{A} \rangle \).
3. If \( \mathbf{A} = \begin{pmatrix} 0 & 0 & a \\ 1 & 0 & b \\ 0 & 1 & c \end{pmatrix} \), then its centralizer is \( \langle \mathbf{A} \rangle \).
4. The matrix \( \mathbf{A} = \begin{pmatrix} \mathbf{J}_{25} + \mathbf{I}_{25} \\ \mathbf{0} \\ \mathbf{I}_{20} \end{pmatrix} \) is invertible.
5. If \( \deg \mu_\mathbf{A}(x) = 1 \), then there is a scalar \( a \) such that \( \mathbf{A} = a\mathbf{I} \).

Consider the matrix \( \mathbf{M} = \begin{pmatrix} \mathbf{0}_n & \mathbf{J}_{nm} \\ \mathbf{J}_{mn} & \mathbf{0}_m \end{pmatrix} \).

6. Find the size of \( \mathbf{M} \).
7. Compute \( \mathbf{M}^2 \) and \( \mathbf{M}^3 \).
8. Give a basis for \( \langle \mathbf{M} \rangle \).
9. Find \( \mu_\mathbf{M}(x) \) and justify your find.
10. Find \( \mu_\mathbf{M}^2(x) \) and justify your find.

**On Bipartite Graphs.** Let \( \mathbf{A} \) be the adjacency matrix of a graph—so \( \mathbf{A} \) is \((0,1)\) symmetric with 0’s on the main diagonal. A graph is called bipartite if one can partition the vertices into two subsets in such a way that all edges are between vertices where one vertex is from one set and the other is from the other.

1. Suppose \( \mathbf{A} \) is bipartite. If one lists the vertices of one set first and then the others, what does the matrix look like in blocks?
2. What would the adjacency matrix of the complete bipartite would look like?
3. Show that if \( \mathbf{A} \) is bipartite then every odd power of \( \mathbf{A} \) has 0’s on the main diagonal.

**Bonus:** Prove the converse of 3.

**On Minimum Polynomials.**

1. What is the minimum polynomial of \( a\mathbf{J}_n \)?
Let \( M = \begin{pmatrix} 0 & J_{3\times4} & 0 \\ 0 & 0 & J_{4\times5} \\ J_{5\times3} & 0 & 0 \end{pmatrix} \). Compute \( M^2 \) and \( M^3 \).

3. Find \( \mu_M(x) \).

4. Find \( \mu_M(x) \) and argue your case.