Consider the veracity or falsehood of each of the following statements. For bonus, argue for those that you believe are true while providing a counterexample for those that you believe are false.

1. \( \varphi(n) \) is even for all \( n > 3 \).
2. There are infinitely many primes congruent to 1 mod 4.
3. If \( \varphi(n) = 2 \) then \( n = 4 \) or \( n = 6 \).
4. Of the odd primes less than 100, half of them are 1 mod 4 and half of them are 3 mod 4.
5. There exist 3 consecutive integers each divisible by a perfect square other than 1.

Given \( |a|_m = 200 \), find \( |a^2|_m, |a^6|_m, |a^{11}|_m \) and \( |a^{15}|_m \).

On 6,718,439. Let \( n = 6,718,439 \).

1. Compute \( \tau(n) \).
2. Compute \( \varphi(n) \).
3. For each divisor \( d \) of \( n \), compute \( \varphi(d) \), and verify Gauss Corollary:
   \[ n = \sum_{d \mid n} \varphi(d) \]
4. Verify that \( \varphi(n) = \tau(n) \cdot \mu(n) \).

Alex claims (correctly) that \( p = 16035002279 \) is a prime. Alex noticed that \( q = 32070004559 \) divides \( 2^p - 1 \) (which it does). Alex concluded \( q \) is a prime. Prove Alex is correct.