Consider the veracity or falsehood of each of the following statements. For bonus, argue for those that you believe are true while providing a counterexample for those that you believe are false. Let $A, B$ and $C$ be lattice points in the plane.

1. If a line $\ell$ contains $A$, then it has infinitely many lattice points.
2. If $\ell$ contains $B$ and $C$ then it contains infinitely many lattice points.
3. The number of nonzero lattice points between the origin and $A$ (including $A$) is the g.c.d of the coordinates of $A$.
4. There are as many lattice points between $A$ and $B$ as between $A - C$ and $B - C$.
5. There are no lattice points strictly between $102,103$ and $10808,11718$.

Find the smallest positive solution to the equation $999x - 49y = 5000$.

Consider the line $5000x + 3339y = 310323$.

1. Find the approximate distance between the $x$-intercept and the $y$-intercept of the line.
2. Find the distance between lattice points of this line.
3. How many nonnegative solutions do you expect? Find them all!
4. Find as small as possible a positive integer $c$ such that the equation $5000x + 3339y = c$ has at least two different positive solutions and find all such solutions.

Alex went into a store and spent one-half of the money Alex had. When Alex came out Alex realized that the money Alex had consisted of as many cents as dollars when going in and half as many dollars as Alex had cents when going in. How much money did Alex have when entering the store?

**Bonus:** Consider the equation $10x + 7y = c$ and find all $c$'s for which there is no nonnegative solutions. Verify the three claims in Sylvester’s Theorem:

1. Every value $c \geq (a-1)(b-1)$ has at least one nonnegative solution;
2. Half of the values below $(a-1)(b-1)$ do not have solutions while for every element of the other half there is exactly one solution;
3. The sum of those values of $c$ for which there is no nonnegative solution is exactly $\frac{(a-1)(b-1)(2ab-a-b-1)}{12}$. 