Consider the veracity or falsehood of each of the following statements. For bonus, argue for those that you believe are true while providing a counterexample for those that you believe are false.

1. If $A$ and $B$ are $2 \times 2$, then $(A - B)^2 = A^2 - 2AB + B^2$.
2. If the first and third rows of $A$ are equal, then the first and third rows of $AB$ are also equal.
3. If the first and third rows of $B$ are equal, then the first and third rows of $AB$ are also equal.
4. $((A^T)^T)^T = A^T$.
5. If two columns of $B$ are equal, then two columns of $AB$ are also equal.

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1. Write the $4 \times 5$ matrix $A$ whose $i, i+1$ entry is $(i-1)^2$ for $i = 1, ..., 4$, and has zeroes everywhere else.
2. Write the $4 \times 5$ matrix $B$ whose $i, j$ entry is $j - i$ for $i = 1, ..., 4$, $j = 1, ..., 5$.
3. Of the following, compute the ones that are possible—the ones that are not state why they are not: $AB$, $AA^T$, $A^TA$ and $AB^T$.

David is an independent supplier of restaurant supplies. He needs to supply three different customers with vegetables and with meat. The first customer, Mr. Smith requires 20 crates of vegetables and 50 pounds of meat, the second, Ms. Jones, requires 18 crates of vegetables and 60 pounds of meat, while the third one, Mr. Colt requires 30 crates of vegetable and 30 pounds of meat.

1. Store this information in a $2 \times 3$ matrix, making clear what your rows and columns stand for.

David can buy the supplies at 4 different wholesalers, Pavilions (P), Vons (V), Albertson’s (A) and Ralph’s (R). Pavilions charges $12 for a crate of vegetables and $45 for pound of meat. Similarly, Vons charges $15 and $40 respectively for vegetables and meat, Albertson’s prices are $18 and $37 for vegetables and meat while those of Ralph’s are $26 and $34 respectively.

2. Store this information in a $4 \times 2$ matrix, making clear what your rows and columns stand for.

For each of the three orders, David is going to order both vegetables and meat from only one of the four suppliers (in order to get free delivery).

3. Multiply the two matrices you have obtained so far.
4. Use the product in 3 to decide who David should buy his supplies from for Mr. Smith’s, and do the same for Ms. Jones and Mr. Colt.

On Digraphs.

1. Let $M$ be the adjacency matrix of the directed 5-cycle:
   Compute $M^2$, $M^3$, $M^4$, and $M^5$.
2. Compute $M^{2015}$.
3. Give a digraph whose adjacency matrix $N$ satisfies $N^4 = 0$, but $N^5 = 0$. 
Bonus: Find a matrix $A$ such that $A^{19} \neq 0$, but $A^{20} = 0$.

Let $J_{m \times n}$ be the $m \times n$ matrix all of whose entries are 1, but as with others $J_m$ denotes the $m \times m$ case. Do the following:

1. Write $2J_3$.
2. Write $2I_3 + 3J_3$.
3. Compute $J_3^2$, $J_3^3$ and $J_3^2$.
4. Compute $p(J_6)$ where $p(x) = x^2 + x + 1$.

Extra Problems.

Let $A = \begin{pmatrix} 1 & 2 & 3 & 4 \\ 5 & 6 & 7 & 8 \\ 9 & 10 & 11 & 12 \\ 13 & 14 & 15 & 16 \end{pmatrix}$. Write it in the form $A = \begin{pmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{pmatrix}$ where each submatrix is $2 \times 2$.

For $A$ in $2 \times 2$ block form $A = \begin{pmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{pmatrix}$, write $A^T$ in $2 \times 2$ block form.

In a survey of a 1000 college students, the following information was obtained: 400 were business majors, of which 50% were female, 300 were engineering majors of which 75% were male, and the remaining were education of which 60% were female. Capture this information using a $2 \times 3$ matrix were the rows are female and male respectively.

On Digraphs. Consider the directed graph with picture. Do the following:

1. Give its adjacency matrix $A$.
2. Use the picture to compute $A^2$.