Homework #1

1. In the ancient city of Nineveh, two merchants (a husband and wife) are discussing the virtues of the number 2 versus the number 3. They are interested in using the balance. The husband claims that if he has weights of 1 libra (ancient unit of weight), 2 libra, 4 libra and 8 libra (the first four powers of 2), he could weigh any object from 1 to 15 libra.
   ① Show that the husband’s claim is correct by showing how one could accomplish it.

But the wife scoffs and responds that if she had just the first three powers of 3, namely, weights of 1 libra, 3 libra and 9 libra, she could accomplish almost the same thing. For example, she could weigh an object of 2 libra by placing it together with the 1 libra weight in one tray and weigh them against the 3 libra weight.
   ② Find all the weights the wife can weigh by just using the weights of 1 libra, 3 libra and 9 libra and justify your answer.

The wife continues by claiming that if she had the first four powers of 3, she could go a lot further than the 15 libra the husband could accomplish with the first four powers of 2.
   ③ Find all the weights the wife could weigh with weights of 1, 3, 9 and 27.

Bonus: Suppose the wife had weights of 1, 3, 9, \ldots, 3^n — what weights could be weighed then?

2. On Old British Money. In Great Britain, from the Norman Conquest of 1066 until 1971, money was as follows: \(d\) stood for pennies or pence (the symbol is for the Roman word denarius), \(s\) stood for shilling and \(£\) for pound (for the Latin word for Libra). There were twelve pennies to a shilling, and twenty shillings to a pound. Do NOT convert everything to pence when doing the following problems and show your work.
   ① If Tommy has 3 £, 18 s and 11 d, and Jimmy has 2 £, 13 s and 9 d, how much money do they have combined?
   ② If Jimmy pays a debt to Tommy of 1 £, 14 s and 11 d, how much money will Jimmy have left over?
   ③ How much money does Tommy have now?
   ④ Seven friends each pay Tommy 10 s, 9 d. How much money did he collect from the 7 friends?
   ⑤ Tommy is going to distribute his money equally among his 5 children. How much does each child get?
   ⑥ If Tommy takes out a loan of 100 £ at 6% APR but compounded monthly, how much money (to the closest penny) will he owe after 4 months?
   ⑦ A rower has a choice of rowing the same distance in a round trip on a placid lake or on a river. Does it matter what he chooses? Elaborate on your answer.