Homework #7

1. According to Ptolemy, \( \text{chord} \, 1^\circ = 1:2,50 \). Using the perimeters, to what approximation of \( \pi \) does this lead to? Is it an overestimate or an underestimate?

2. We saw that the Hindus made a colossal contribution to trigonometry by switching from chords to half-chords (sines and cosines). Many centuries later (18th century) another major contribution would be made when the switch from degrees to radians would be executed. Radians, which are eminently necessary for simplicity in calculus calculations, are also useful for other reasons such as making some inequalities symbolically much simpler.

   ① Use the fact that the straight line is the shortest distance between two points, and the following picture to prove that \( \sin \alpha \leq \alpha \) for any \( 0 \leq \alpha \leq \pi/2 \) (so \( \alpha \) is measured in radians).

   ② Use Archimedes’ assumptions about convex curves (page 79), and the following picture to prove that \( \alpha \leq \tan \alpha \) for any \( 0 \leq \alpha \leq \pi/2 \) (so \( \alpha \) is measured in radians).

3. A powerful unvanquished excellent black snake which is 80 angulas in length enters into a hole at the rate of \( \frac{71}{2} \) angulas in \( \frac{5}{14} \) of a day, and in the course of \( \frac{1}{4} \) of a day its tail grows \( \frac{11}{4} \) of an angula. O ornament of arithmeticians, tell me by what time this serpent enters fully into the hole?

4. Prove the following algebraic identity

\[
\sqrt{x + \sqrt{y}} = \sqrt{\frac{x + \sqrt{x^2 - y}}{2}} + \sqrt{\frac{x - \sqrt{x^2 - y}}{2}}
\]