1. Consider the polynomial \( p(z) = z^3 - 4z^2 + 5z - 2 \) and draw the curves \( U \) and \( T \). Hint: Use Desmos.

2. The set \( \mathbb{Z}[i] = \{ a + bi \mid a, b \in \mathbb{Z} \} \) is known as the Gaussian integers. Let \( z, w \in \mathbb{Z}[i] \) be such. One says that \( w \mid z \) (\( w \) divides \( z \)) if \( \frac{z}{w} \in \mathbb{Z}[i] \).
   - If \( z = a + bi \) is a Gaussian integer, compute \( z \bar{z} \).
   - Show that if \( w \mid z \), then \( (w \bar{w}) \mid (z \bar{z}) \).
   - Find all Gaussian integers that divide 1.
   - Decide true or false: \( 1 + i \mid 2 \), \( 2 + i \mid 5 \), \( 1 + i \mid 7 \).

A Gaussian integer is called prime if every factorization is trivial.

3. More Sum of Squares and the Quaternions. Consider the following set of \( 2 \times 2 \) complex matrices: \( \mathcal{Q} = \left\{ \begin{pmatrix} \alpha & \beta \\ -\bar{\beta} & \bar{\alpha} \end{pmatrix} \mid \alpha, \beta \in \mathbb{Z}[i] \right\} \) where \( \bar{\alpha} \) is the conjugate of \( \alpha \). An element \( M \in \mathcal{Q} \) is called a (n integral) quaternion. Do the following:
   - Prove that the sum and the product of two elements of \( \mathcal{Q} \) is an element of \( \mathcal{Q} \).
   - Compute \( \det M \) for any quaternion and show that it is never 0 unless \( M \) is zero.
   - Find two quaternions that do not commute (under multiplication of course).
   - Assume that every prime is the sum of four integer squares, prove that every positive integer is the sum of four squares (possibly 0 of course).
   - Find a quaternion \( M \) so that \( \det M = \# \) where \( \# \) is your student id number. Hint: Be greedy.

Bonus: Find a number that cannot be written as a sum of 8 cubes (Waring).