Homework # 3

1. We are going to study the counting of heads in the tossing of a fair coin. We are going to let \( X_n \) denote the random variable that denotes the number of heads in \( n \) flips. Decide which of the following statements is true or false and give reasons for your answers.

\[ P( X_2 = 1) = 0.5 . \]

\[ P( X_{10} = 5) = 0.5 . \]

\[ \lim_{n \to \infty} P( n - 50 \leq X_{2n} \leq n + 50) = 1 . \]

\[ \lim_{n \to \infty} P( X_{2n} = n) = 0 . \]

\[ \lim_{n \to \infty} P(0.99n \leq X_{2n} \leq 1.01n) = 1 . \]

\[ 0 < \lim_{n \to \infty} \frac{P( X_{2n} = n)}{\sqrt{n}} < 1 . \]

\[ E( X_{2n}) = n . \]

2. Although not available for another century after Bernoulli, besides the expectation of Huygens, another parameter became of huge importance—the standard deviation and its square called the variance. It is easier to define the variance \( V(X) \) is simply

\[ V(X) = E(X^2) - E(X)^2 . \]

2. This problem is based on a true historical situation, so for the time being assume you are in Italy in the late 1500’s. As your inheritance, your parents will purchase shares on ships that travel the seas. They have promised to buy you a total of four shares in any one ship or a collection of ships that you choose.

From past evidence, you have gathered that one share from any returning ship is worth 10 ducats if the ship comes back from its travels, but that unfortunately only half the ships do so. Assume different ships move across the sea independently of one another. Let \( H \) be the random variable which is your inheritance.

1. Suppose you choose to buy your four shares in one ship.
   1. Find the probability distribution for \( H \).
   2. Compute \( E(H) \) and \( V(H) \).
   3. How many standard deviations is the mean away from ruin?

2. Answer the same four questions 1, 2, and 3 as in 1 if you now choose to buy two shares in one ship and two shares in another.

3. Answer the same four questions 1, 2, and 3 as in 1 if you now choose to buy one share in each of four different ships.

4. Which is the better investment scheme? Write a few sentences in support of your decision.
You work for a mail-order company that advertises that it ships 90% of its orders within three-working days. You randomly select 100 orders out of the more than 5000 orders from last week’s process. You will consider the random variable \( Y \) that counts the number of shipments within three days out of the random 100 orders chosen.

1. Assuming that the company’s claim is correct, what random variable should you use to compute with for \( Y \)?
2. What should you expect for \( Y \), \( E(Y) = ? \).
3. What is \( V(Y) \) and how shocked would you be if \( Y = 86 \)?

**Bonus.** Give a value of \( Y \) that will compel you to report your company.