Math 760 Day 8 (2-7)

Uniqueness of prime decompositions

\( M \cong P_1 \# \cdots \# P_k \# \mathbb{R} \times S^2 \)
\( \cong Q_1 \# \cdots \# Q_m \# n(S^1 \times S^2) \)

Let \( S \) be a collection of \( \mathbb{Z} \)-spheres s.t.

\( M \# S \) is \( n \)-punctured \( P_1 \) and punctured \( S^3 \)

\( M - T \) is \( n \)-punctured \( Q_1 \) and punctured \( S^3 \)

If lost time we can rechoose \( S \)

s.t. \( TNS = \emptyset \).

Then \( M - (TNS) \) is the union of

- \( n \)-punctured \( P_i \) and punctured \( S^3 \)
- \( \mathbb{Q} \) and punctured \( S^3 \)

Hence \( k = m \) and \( \exists P_1, \ldots, P_k, Q_1, \ldots, Q_m \)

To show \( l = n \), note

\( M \# N \# (S^1 \times S^2) \cong N \# m(S^1 \times S^2) \)

\( H_1(M) \cong H_1(N) \# \mathbb{Z}^2 \cong H_1(N) \# \mathbb{Z}^n \)

so \( l = n \).
Introduction to 2-fold branched covers.

Def: An **trivial tangle** is an embedding of $0 \subseteq [0,1]$ into $\mathbb{R}^3$ ambient isotopic to $n$ strands.

Def: A **handlebody** is a compact 3-manifold homeomorphic to the closed regular nbh of a finite graph embedded in $\mathbb{R}^3$.

Def: A 2-fold branched cover...

Ex1 The genus $n$ handlebody 2-fold branched covers the $n+1$ strand trivial tangle.

Ex1
**Def:** A Heegaard splitting of a closed 3-manifold \( M \) is a triple \((H_1, H_2, f)\) s.t. \( H_1 \) and \( H_2 \) are handlebodies, \( f \) is a homeomorphism \( f : \partial H_1 \to \partial H_2 \) s.t. \( M = H_1 \cup H_2 \) where \( x \sim y \) if \( x \in \partial H_1 \) and \( y \in \partial H_2 \) and \( f(x) = y \).

**Ex:** \( L(p,q) \) lens space.

\[ L(p,q) \cong D^2 \times S^1 \cup D^2 \times S^1 / \sim \]

**Ex:**

\[ S^3 \cong \begin{array}{c}
\bigcirc \bigcup \bigcirc \\
B^3 \\
B^3 
\end{array} \]

**Def:** A bridge splitting of a knot \( K \) in \( S^3 \) is a triple \((T_1, T_2, f)\) s.t. \( T_1 \) and \( T_2 \) are trivial tangles and \( f \) is an orientation reversing homeomorphism \( f : \partial T_1 \to \partial T_2 \) s.t. \( (S^3, K) \cong T_1 \cup T_2 \) where \( x \sim y \) if \( x \in \partial T_1 \) and \( y \in \partial T_2 \) and \( f(x) = y \).

**Ex:** bridge surface