

## MATH 550, HOMEWORK 1

### DEFINITION OF TOPOLOGY, OPEN SETS, AND CLOSED SETS

**Due end of day, Thursday, Sept. 6th** Note that problems marked with a Q have appeared on past comprehensive exams.

Reading. Read §12, §13 of Munkres.

Problems.

- (1) Munkres §13 exercise 6.
- (2) Munkres §13 exercise 8. (You may use standard facts about the real and rational numbers.)
- (3) Q: Let  $(X, \tau)$  be a topological space and let  $D \subset X$ . Prove that  $\overline{D} = X$  if and only if  $D$  has non-trivial intersection with every non-empty element of  $\tau$ .
- (4) Q: Let  $\tau$  denote the collection of the empty set and all subsets of  $\mathbb{R}$  that contain the element 0.
  - (a) Prove  $\tau$  is a topology on  $\mathbb{R}$ .
  - (b) Find the interior of the rational numbers,  $\mathbb{Q}$ , as a subset of  $(\mathbb{R}, \tau)$ .
  - (c) Find the interior of  $(1, 5)$  as a subset of  $(\mathbb{R}, \tau)$ .
  - (d) Find the closure of  $[-1, 2]$  as a subset of  $(\mathbb{R}, \tau)$ .
  - (e) Find the closure of  $(1, 5)$  as a subset of  $(\mathbb{R}, \tau)$ .