Math 550A, Homework 9

Separation Axioms and Urysohn’s Lemma

Due in class on May 6th

Exercises (to do on your own)

1. Prove that $\mathbb{R}_K$ is $T_2$ but not $T_3$. (Recall $\mathbb{R}_K$ from §13. Hint: $K$ is a closed set in the $\mathbb{R}_K$ topology.)

2. Let $X$ and $X'$ denote a single set under two different topologies. Suppose $X'$ is finer than $X$. If $X$ is Hausdorff, must $X'$ be Hausdorff? Answer the same question for “$T_1$”, “regular” and “normal” in place of “Hausdorff.”

3. Prove that if $X$ is regular, then every pair of distinct points in $X$ have neighborhoods whose closures are disjoint.

Problems (to be turned in)


2. Munkres, §32, exercise 1. (Note this can fail if we don’t assume the subspace is closed – see example 1 of this section.)

3. Prove the Urysohn Lemma directly for a metric space $(X, d)$. (Hint: begin by defining the distance $d(x, A)$ between a point $x$ and a set $A$, and showing $d(x, A)$ is continuous as a function of $x$.)