Math 500, Homework 7

Covering spaces, fundamental group

Due at start of class, Thursday, 12/8

Reading §53-55

Exercises (to do on your own)

1. Find a covering $p : E \to B$ of the figure-eight B such that $p^{-1}(b)$ consists of three points for each b.

2. Let X be the figure-eight space, and let x_0 be the "crossing point" in the middle. Convince yourself (without rigorous proof) that $\pi(X, x_0)$ is non-abelian. (This means there exist $[f], [g] \in \pi_1(X, x_0)$ such that $[f] * [g] \neq [g] * [f]$.)

3. Let G and H be groups. Prove that the operation on $G \times H$ defined by

$$(g_1, h_1) \cdot (g_2, h_2) = (g_1 \cdot g_2, h_1 \cdot h_2)$$

makes $G \times H$ into a group.

4. Convince yourself that the Brouwer fixed point theorem could fail if you replaced B with the open disk

$$B^\circ = \{(x, y) \in \mathbb{R}^2 : x^2 + y^2 < 1\}.$$

Problems (to turn in)

1. Let $p : E \to B$ be a covering map. Given $b \in B$, prove that $p^{-1}(\{b\})$ is a discrete subspace of E (i.e., the subspace topology is the discrete topology).

2. Prove that a covering map $p : E \to B$ is an open map. (Being also surjective and continuous, it follows that p is a quotient map.)

3. Let X and Y be spaces, with $x_0 \in X$ and $y_0 \in Y$. Prove that

$$\pi_1(X \times Y, x_0 \times y_0) \cong \pi_1(X, x_0) \times \pi_1(Y, y_0)$$

as groups. (Exercise 3 explains the meaning of the product of two groups.)