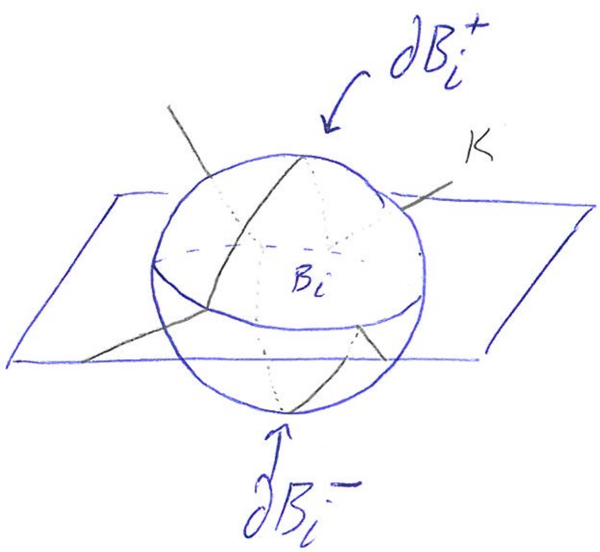
Knot theory Day 24

<u>Thm</u> (Menasco) If K is a prime, alternating knot, then K is <u>not</u> a satellite knot.

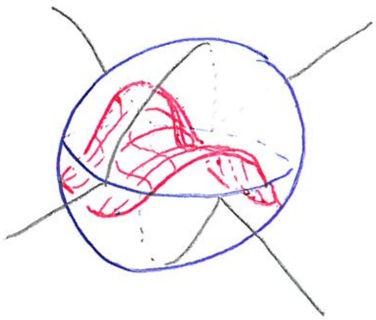
Assume K is a prime, alternating satellite knot to arrive at a contradiction.

Let T be the companion torus for K. We ~~should~~ showed T is incompressible last time.

Menasco form

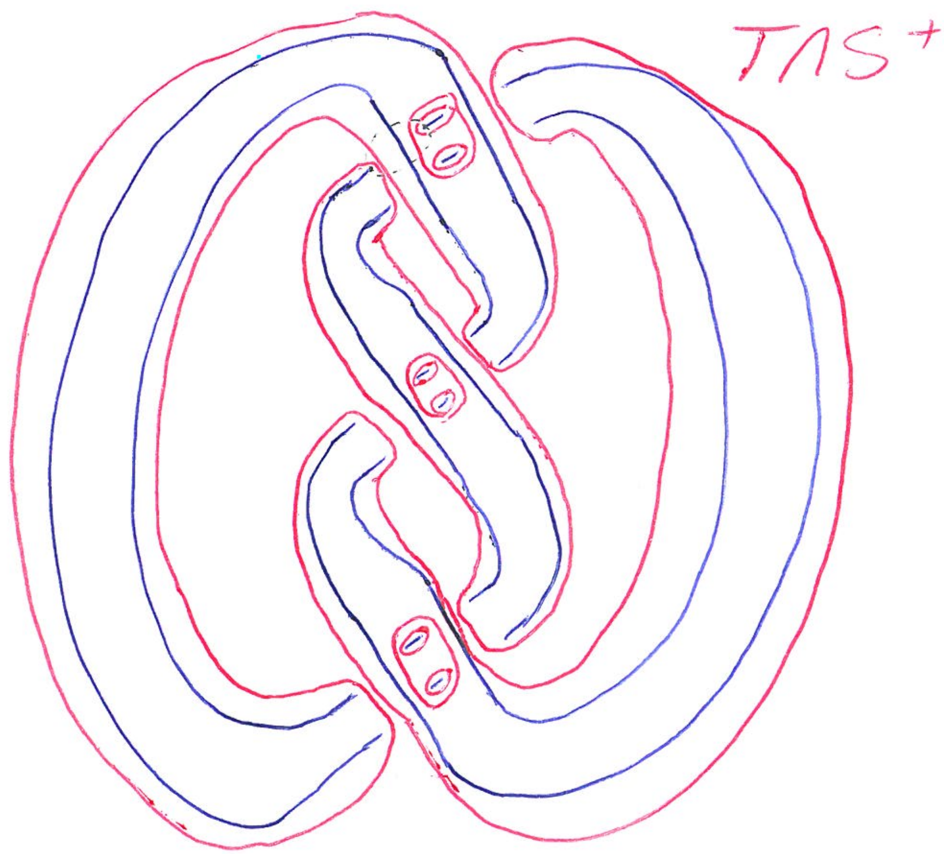


- $S =$ sphere of projection
- $B_i =$ 3-ball "baubles" at each crossing
- K imbedds in $S \cup (\bigcup_{i=1}^{n} \partial B_i)$.
- $S^+ = (S - (\bigcup_{i=1}^{n} B_i)) \cup (\bigcup_{i=1}^{n} \partial B_i^+)$



- T meets B_i in a collection of ~~saddles~~ disks deformed to look like saddles

- T meets S^+ in exactly a diagram of K



Assume we have isotoped T while fixing K such that the following lexigraphicall ordering is minimized $\left(\left|T \cap \left(\bigcup_{i=1}^{n} B_i\right)\right|, |T \cap S^+| + |T \cap S^-|\right)$

- If T minimizes the complexity we say T is minimal.

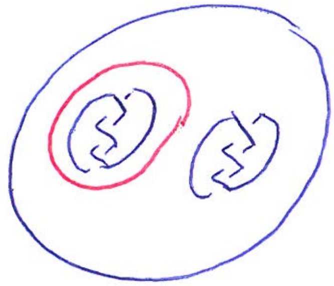
Claim 1 | If T is minimal, then no curve of $T \cap S^+$ is disjiont from $\bigcup_{i=1}^{\infty} B_i$. (Same goes for $T \cap S^-$)

Pf | Suppose $\gamma \subset T \cap S^+$ is a curve s.t.

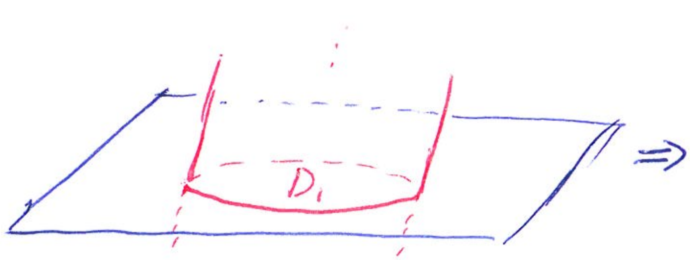
γ is disjiont from $\bigcup_{i=1}^{\infty} B_i$.

By the Schönflies thm, γ separates S^+ into two disks D_1 and D_2

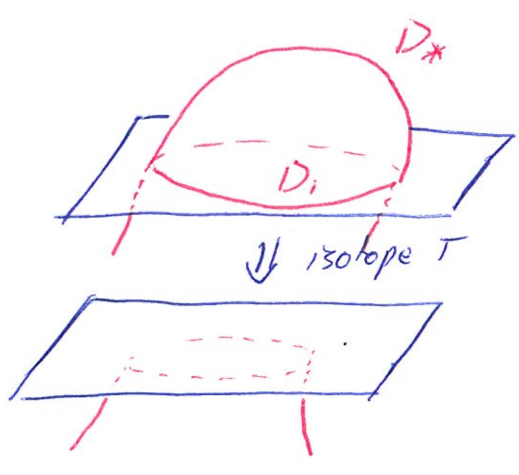
- If $D_1 \cap K \neq \emptyset$ and $D_2 \cap K \neq \emptyset$, then this contradicts the fact that K is a knot



- Hence we can assume $D_1 \cap K = \emptyset$ and that γ is innermost. ~~Then D_1~~ If ∂D_1 is essential in T, then D_1 is a compressing disk for T. ✱ Hence, γ bounds a disk D_* in T.



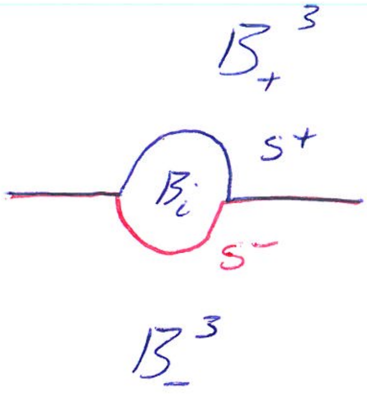
$\Rightarrow$



Since $D_* \cup D_1$ is a 2-sphere in $S^3 - K$, then isotope D_* past D_1 eliminating γ as a loop of intersection.

Let B^3_+ be the 3-ball "above" S^+ in S^3.

Let B^3_- be the 3-ball "bellow" S^- in S^3.



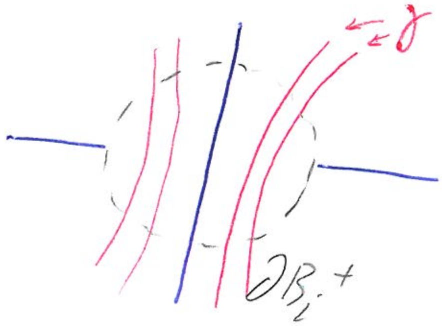
Claim 2| If T is minimal, $T \cap B^3_+$ and $T \cap B^3_-$ is a collection of disks.

Pf| Fact: Every surface in B^3 other than a disk is compressible. Hence, if a component of $T \cap B^3_+$ is not a disk then we contradict incompressibility of T (this is a little bit of a lie, but OK for now).

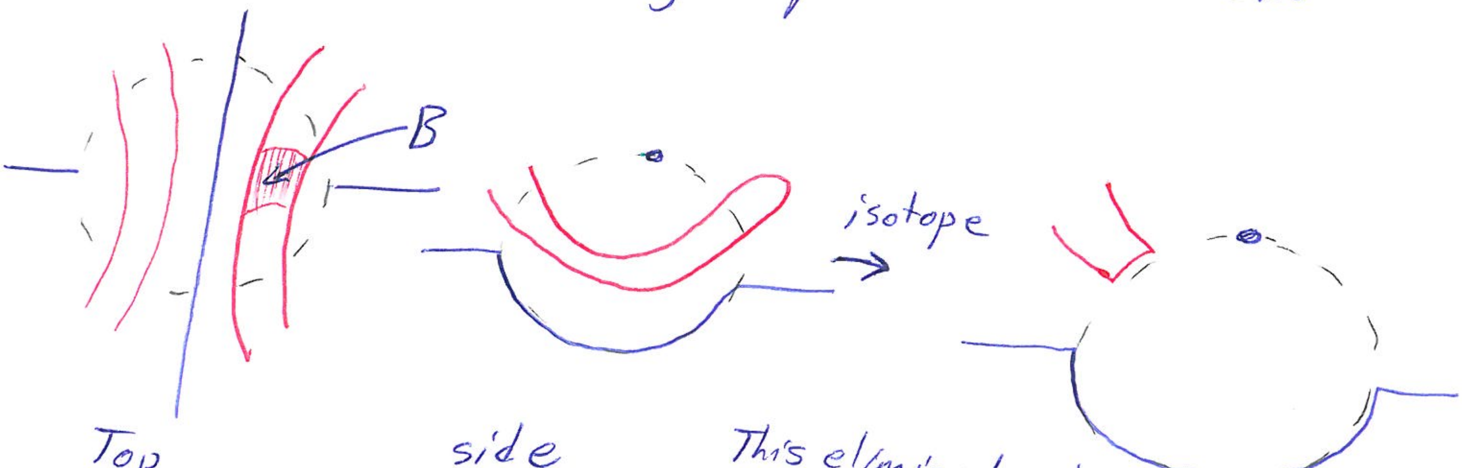
Claim 3| For each i, ~~no~~ If T is minimal, no curve of $T \cap S^+$ meets ∂B_i^+ twice.

Pf| Assume $\gamma \subset T \cap S^+$ meets ∂B_i^+ twice to arrive at a contradiction.

Case 1

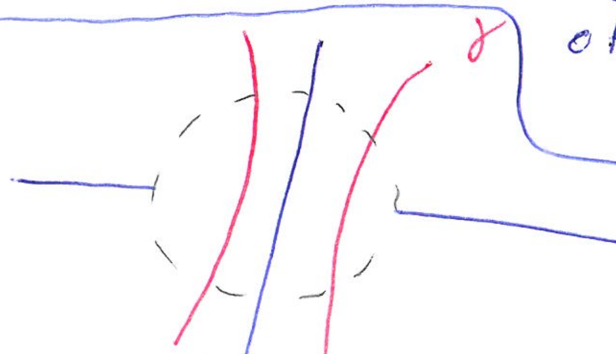


Remember, by Claim 2 γ bounds a disk D in B^3_+. Let B be a band illustrating a portion of this disk.



This eliminates two components of $|T \cap (\bigcup_{i=1} \partial B_i)|$

Case 2

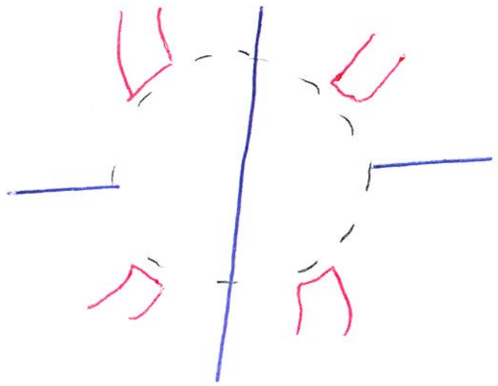


Top

γ bounds a disk D in B^3_+. Let B be a band illustrating a portion of D.

Hence, there is a meridian disk for T that meets K in one point. This contradicts K is prime.

In case 1, after the isotopy the top view is



In either case, we arrive at a contradiction. Hence, every curve of $T \cap S^+$ meets any ∂B_i^+ in at most one arc. $\square$