Math 123: Series Convergence Tests III

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Tuesday April 5, 2016

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Conditional and Absolute Convergence

Definition

A series $\sum_{i=1}^{\infty} a_i$ is **absolutely** convergent if $\sum_{i=1}^{\infty} |a_i|$ is convergent.

Definition

A series $\sum_{i=1}^{\infty} a_i$ is **conditionally** convergent if $\sum_{i=1}^{\infty} |a_i|$ is divergent and $\sum_{i=1}^{\infty} a_i$ is convergent.

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Determine conditional or absolute convergence of $\sum_{i=1}^{\infty} \frac{\cos(i)}{i^2}$.

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Ratio Test

Theorem

Given a series $\sum_{i=1}^{\infty} a_i$. If

$$\lim_{i\to\infty}|rac{a_{i+1}}{a_i}|=L,$$

then

- If L < 1, the series converges absolutely.
- **2** If L = 1, the test is inconclusive.
- **3** If L > 1, the series diverges.

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Does not help for the mundane: $\sum_{i=1}^{\infty} \frac{1}{i^2}$ Helps with the crazy stuff: $\sum_{i=1}^{\infty} \frac{i^i}{i!}$

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Root Test

Theorem

Given a series $\sum_{i=1}^{\infty} a_i$. If

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If L < 1, the series converges absolutely.
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Given a series $\sum_{i=1}^{\infty} a_i$. If

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then

• If L < 1, the series converges absolutely. 2 If L = 1, the test is inconclusive. \bullet If L > 1, the series diverges.

Helps with series involving variable powers: $\sum_{i=1}^{\infty} (\frac{i}{3i+4})^i$

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Helps with series involving variable powers: $\sum_{i=1}^{\infty} \left(\frac{i}{3i+4}\right)^i$ Helps with series involving variable powers: $\sum_{i=1}^{\infty} i\left(\frac{2}{3}\right)^i$

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