Math 123: Operations on Power Series

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1 Power Series
Definition

A **Power Series** is a series and a function of the form

\[ P(x) = \sum_{k=0}^{\infty} c_k (x - a)^k = c_1 + c_2(x - a) + c_3(x - a)^2 + \ldots \]

The radius of convergence is a positive number \( R \) such that \( P(x) \) converges for \( x \) such that \( |x - a| < R \).

\[ R = \lim_{k \to \infty} \left| \frac{c_k}{c_{k+1}} \right| \]
Interval of Convergence

Given a power series $\sum_{k=0}^{\infty} c_k (x - a)^k$ with radius of convergence $R$, the **interval of convergence** is one of the following where we include endpoints if the series is convergent at those points.

$$(a - R, a + R), [a - R, a + R), (a - R, a + R], [a - R, a + R]$$
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**Exercise:** Find the interval of convergence for the power series $\sum_{k=1}^{\infty} \frac{(x)^k}{k}$. 
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**Exercise:** Find the interval of convergence for the power series $\sum_{k=1}^{\infty} \frac{(x)^k}{k}$.

**Exercise:** Find the interval of convergence for the power series $\sum_{k=1}^{\infty} \frac{(3x-3)^k}{k^2 5^k}$.
Using the geometric series

Exercise: Use

\[
\frac{1}{1-x} = \sum_{n=0}^{\infty} x^n
\]

to find a power series for \( f(x) = \frac{1}{1+x^2} \) and find the interval of convergence.
Theorem

If \( P(x) = \sum_{k=0}^{\infty} c_k (x - a)^k \), then

\[
P'(x) = \sum_{k=1}^{\infty} kc_k (x - a)^{k-1}
\]

\[
\int P(x) \, dx = C + \sum_{k=0}^{\infty} \frac{c_k}{k+1} (x - a)^{k+1}
\]
Derivatives and Integrals of Series

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Exercise: Find the derivative of \( \sum_{n=0}^{\infty} \frac{x^n}{n!} \).
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Exercise: Find the derivative of \( \sum_{n=0}^{\infty} \frac{x^n}{n!} \).

Exercise: Find the power series for \( f(x) = \tan^{-1}(x) \).
Derivatives and Integrals of Series

Theorem

If $P(x) = \sum_{k=0}^{\infty} c_k (x - a)^k$, then

$$P'(x) = \sum_{k=1}^{\infty} kc_k (x - a)^{k-1}$$

$$\int P(x) \, dx = C + \sum_{k=0}^{\infty} \frac{c_k}{k+1} (x - a)^{k+1}$$

**Exercise:** Find the derivative of $\sum_{n=0}^{\infty} \frac{x^n}{n!}$.

**Exercise:** Find the power series for $f(x) = \tan^{-1}(x)$.

**Exercise:** Find the power series for $f(x) = \ln(1 + x)$. 

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