Math 123: Linear D.E.s of First and Second Order

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1 First Order Differential Equations

2 Integrating Factor Method

3 Superposition Principle
Types of Differential equations

Definition
A first order linear D.E. is of the form

\[ y' + Q(x)y = R(x) \]

where \( Q(x) \) and \( R(x) \) are functions of \( x \).
Types of Differential equations

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A first order **linear** D.E. is of the form

\[ y' + Q(x)y = R(x) \]

where \( Q(x) \) and \( R(x) \) are functions of \( x \).

Definition
A second order **linear** D.E. is of the form

\[ y'' + P(x)y' + Q(x)y = R(x) \]

where \( P(x) \), \( Q(x) \) and \( R(x) \) are functions of \( x \).
Types of Differential equations

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A first order **linear** D.E. is of the form

\[ y' + Q(x)y = R(x) \]

where \( Q(x) \) and \( R(x) \) are functions of \( x \).

Definition
A second order **linear** D.E. is of the form

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where \( P(x) \), \( Q(x) \) and \( R(x) \) are functions of \( x \).

If \( R(x) = 0 \) we call the D.E. **homogeneous**.
Integrating Factor Method

**Question 1:** Given a D.E. $y' + Q(x)y = R(x)$, if you could find a function $f(x)$ such that

$$\frac{d(f(x)y)}{dx} = f(x)(y' + Q(x)y)$$

could you solve the D.E.? 

Answer: Yes!
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Answer: Yes!

Question 2: Given a D.E. $y' + Q(x)y = R(x)$, can you find the formula for a function $f(x)$ such that

$$\frac{d(f(x)y)}{dx} = f(x)(y' + Q(x)y)$$
Integrating Factor Method

**Question 1:** Given a D.E. \( y' + Q(x)y = R(x) \), if you could find a function \( f(x) \) such that

\[
\frac{d(f(x)y)}{dx} = f(x)(y' + Q(x)y)
\]

could you solve the D.E.?

**Answer:** Yes!

**Question 2:** Given a D.E. \( y' + Q(x)y = R(x) \), can you find the formula for a function \( f(x) \) such that

\[
\frac{d(f(x)y)}{dx} = f(x)(y' + Q(x)y)
\]

**Answer:** Yes! \( f(x) = e^{\int Q(x)dx} \)
Integrating Factor Method

To solve $y' + Q(x)y = R(x)$,

1. Multiply both sides by $f(x) = e^{\int Q(x)dx}$
2. Recognize that the L.H.S. is $\frac{d(f(x)y)}{dx}$
3. Integrate both sides and solve for $y$.

Exercise: Solve $y' + 2y = 2e^x$.
Exercise: Solve $xy' + y = \sqrt{x}$. 
Superposition Principle

**Theorem**

*Given a homogeneous linear differential equation with solutions* \( f(x) \) *and* \( g(x) \) *then* \( a \cdot f(x) + b \cdot g(x) \) *is also a solution for any constants* \( a \) *and* \( b \).

**Exercise:** Demonstrate this theorem for the D.E.

\[ y'' + P(x)y' + Q(x)y = 0 \]