Math 123: Introduction to Differential Equations

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1. Definition of Differential Equation
2. Models for Population Growth
3. Separable Differential Equations
Definition

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The Math of Frisky Bunnies

Suppose bunnies reproduce according to the following rules

1. We start in month zero with one male and one female bunny.
2. Every month each female bunny gives birth to one male and one female bunny.

Let \( B(t) \) be the number of bunnies \( t \) months after month zero.
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Let $B(t)$ be the number of bunnies $t$ months after month zero.
Estimate $B'(t)$.
How should we model $B(t)$?
A Few Famous Differential Equations

1. Einstein’s field equation in general relativity
2. The Navier-Stokes equations in fluid dynamics
3. Verhulst equation - biological population growth
4. The Black-Scholes PDE - models financial markets
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\frac{dy}{dx} = f(x)g(y)
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Any solution to the following integral equation is a solution to the above differential equation.

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\int \frac{1}{g(y)} dy = \int f(x) dx
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**Example:** Solve the following D.E. \( \frac{dy}{dx} = y \).
Logistic Growth (the Verhulst model)

Hypotheses for the population model:

1. For small populations the population growth is proportional to the population size.
2. The population can not grow larger than a carrying capacity $M$. 

These hypotheses give rise to the following D.E.

$$\frac{dy}{dx} = ky(M-y)$$

Where $k$ and $M$ are constants.

Exercise: Find the general solution to this D.E.
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