## Math 123 Final Exam Spring 2015

NAME (PRINTED):
Please turn off all electronic devices. You may use both sides of a $8.5 \times 11$ sheet of paper for notes while you take this exam. No calculators, no course notes, no books, no help from your neighbors. Show all work - the grading will be based on your work shown as well as the end result. Remember to put your name at the top of this page. Good luck.

| Problem | Score (out of) |
| :---: | ---: |
| $\mathbf{1}$ | $(10)$ |
| $\mathbf{2}$ | $(10)$ |
| $\mathbf{3}$ | $(10)$ |
| $\mathbf{4}$ | $(10)$ |
| $\mathbf{5}$ | $(10)$ |
| $\mathbf{6}$ | $(10)$ |
| $\mathbf{7}$ | $(10)$ |
| $\mathbf{8}$ | $(10)$ |
| Total | $(80)$ |

1. (10 pts) Solve the following IVP

$$
y^{\prime \prime}-4 y^{\prime}+4 y=0 \text { and } y(0)=1, y^{\prime}(0)=1
$$

2. (10 pts) Find the interval of convergence for the following power series.

$$
\sum_{n=1}^{\infty} \frac{(2 x-2)^{n}}{\left(n^{2}\right)\left(6^{n}\right)}
$$

3. (10 pts) Evaluate the following integral

$$
\int \frac{1}{x \sqrt{1-x^{2}}} d x
$$

4. ( 10 pts ) Let $V$ be the volume of the solid obtained by rotating the region bounded by $y=x^{2}$ and $y=x+2$ about the line $x=-1$.
A) Express $V$ as a definite integral using the shell method. DO NOT EVALUATE THE INTEGRAL.
B) Express $V$ as a definite integral using the washer method. DO NOT EVALUATE THE INTEGRAL.
5. (10 pts) Determine if the following series converges or diverges. Carefully justify your answer.

$$
\sum_{n=1}^{\infty}\left(\frac{n^{2} \cdot \tan ^{-1}(n)}{\pi n^{2}+1}\right)^{n}
$$

6. (10 pts) Solve the following D.E.

$$
y^{\prime}+\frac{1}{\tan ^{-1}(x) \cdot\left(x^{2}+1\right)} y=\frac{x \cdot \ln (x)}{\tan ^{-1}(x)}
$$

7. (10 pts) Use Taylor's Estimation Theorem to find a bound on the error in approximating $f(x)=\ln (1+x)$ by the 21st Taylor polynomial centered at $x=0$ on the interval $\left[-\frac{1}{2}, \frac{1}{2}\right]$.
8. (10 pts) Find all values of $\theta$ where the following polar curve has a horizontal tangency.

$$
r=\cos (\theta)
$$

