Math 123: Trig Substitution

Ryan Blair

CSU Long Beach

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Outline

A Motivating Example

Find $\int_{-1}^{1} \sqrt{1-x^2}dx$ in two different ways.

Method One: Geometric.

Method Two: Using Trigonometric identities.

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Method One: Geometric. Since $y = \sqrt{1-x^2}$ is the top half of the unit circle, use definition of integral as area under the curve.

Method Two: Using Trigonometric identities. Make the substitution $x = sin(\theta)$ and use $cos^2(\theta) + sin^2(\theta) = 1$.

Trig Substitution

For integrals involving $\sqrt{a^2 - x^2}$, $\sqrt{x^2 - a^2}$ or $\sqrt{x^2 + a^2}$ where a is a constant, we can often integrate by constructing a right triangle with one of these values as a side length.

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Method Three: Find $\int_{-1}^{1} \sqrt{1 - x^2} dx$ by building the relevant right triangle and making a substitution.

Example: Find $\int \frac{1}{x^2\sqrt{x^2+9}}$

Example: Find $\int \frac{1}{\sqrt{x^2-4}}$