Math 123: Trig Substitution

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Outline

1. Trig Substitution
A Motivating Example

Find $\int_{-1}^{1} \sqrt{1 - x^2} \, dx$ in two different ways.

**Method One:** Geometric.

**Method Two:** Using Trigonometric identities.
A Motivating Example

Find \( \int_{-1}^{1} \sqrt{1 - x^2} \, dx \) in two different ways.

**Method One:** Geometric. Since \( y = \sqrt{1 - x^2} \) is the top half of the unit circle, use definition of integral as area under the curve.

**Method Two:** Using Trigonometric identities.
Find $\int_{-1}^{1} \sqrt{1 - x^2} \, dx$ in two different ways.

**Method One:** Geometric. Since $y = \sqrt{1 - x^2}$ is the top half of the unit circle, use definition of integral as area under the curve.

**Method Two:** Using Trigonometric identities. Make the substitution $x = \sin(\theta)$ and use $\cos^2(\theta) + \sin^2(\theta) = 1$. 
For integrals involving $\sqrt{a^2 - x^2}$, $\sqrt{x^2 - a^2}$ or $\sqrt{x^2 + a^2}$ where $a$ is a constant, we can often integrate by constructing a right triangle with one of these values as a side length.
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**Method Three:** Find $\int_{-1}^{1} \sqrt{1 - x^2} \, dx$ by building the relevant right triangle and making a substitution.
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**Method Three:** Find $\int_{-1}^{1} \sqrt{1 - x^2} \, dx$ by building the relevant right triangle and making a substitution.

**Example:** Find $\int \frac{1}{x^2\sqrt{x^2+9}} \, dx$

**Example:** Find $\int \frac{1}{\sqrt{x^2-4}} \, dx$