Outline

1. Trig Integrals

2. Trig Substitution
Review From Last Time

For \( \int \cos^{odd}(x) \, dx \) or \( \int \sin^{odd}(x) \, dx \) use

\[
\cos^2(x) + \sin^2(x) = 1
\]

and u-substitution
Review From Last Time

For $\int \cos^{\text{odd}}(x)\,dx$ or $\int \sin^{\text{odd}}(x)\,dx$ use

$$\cos^2(x) + \sin^2(x) = 1$$

and u-substitution

For $\int \cos^{\text{even}}(x)\,dx$ or $\int \sin^{\text{even}}(x)\,dx$ use

$$\cos^2(x) = \frac{1}{2}(1 + \cos(2x))$$

$$\sin^2(x) = \frac{1}{2}(1 - \cos(2x))$$

possibly multiple times
How do we integrate Trigonometric functions?

Example: \[ \int \sin^2(x) \cos^3(x) \, dx. \]
How do we integrate Trigonometric functions?

**Example:** \( \int \sin^2(x) \cos^3(x) \, dx \).

For \( \int \sin^{\text{anything}}(x) \cos^{\text{odd}}(x) \, dx \) or \( \int \cos^{\text{anything}}(x) \sin^{\text{odd}}(x) \, dx \) use

\[
\cos^2(x) + \sin^2(x) = 1
\]

and u-substitution.
How do we integrate Trigonometric functions?

**Example:** \( \int \sin^2(x)\cos^3(x) \, dx \).

For \( \int \sin^{\text{anything}}(x)\cos^{\text{odd}}(x) \, dx \) or \( \int \cos^{\text{anything}}(x)\sin^{\text{odd}}(x) \, dx \) use

\[\cos^2(x) + \sin^2(x) = 1\]

and u-substitution

**Question:** What about \( \int \sin^{\text{even}}(x)\cos^{\text{even}}(x) \, dx \)
How do we integrate Trigonometric functions?

Example: \( \int \sin^2(x) \cos^3(x) \, dx \).

For \( \int \sin^{\text{anything}}(x) \cos^{\text{odd}}(x) \, dx \) or \( \int \cos^{\text{anything}}(x) \sin^{\text{odd}}(x) \, dx \) use

\[
\cos^2(x) + \sin^2(x) = 1
\]

and u-substitution

**Question:** What about \( \int \sin^{\text{even}}(x) \cos^{\text{even}}(x) \, dx \)?

Use double angle formula lots!
How do we integrate Trigonometric functions?

Example: \( \int \tan(x) \sec^4(x) \, dx \).
How do we integrate Trigonometric functions?

Example: \[ \int \tan(x)\sec^4(x)\,dx. \]

For integrals involving \( \tan(x) \) and \( \sec(x) \) use

\[ 1 + \tan^2(x) = \sec^2(x) \]

and u-substitution.
Some Challenges

Example: Find $\int \sec(x) \, dx$.

Example: Find $\int \sec^3(x) \, dx$.
A Motivating Example

Find $\int_{-1}^{1} \sqrt{1 - x^2} \, dx$ in two different ways.

**Method One:** Geometric.

**Method Two:** Using Trigonometric identities.
Find \( \int_{-1}^{1} \sqrt{1-x^2} \, dx \) in two different ways.

**Method One:** Geometric. Since \( y = \sqrt{1-x^2} \) is the top half of the unit circle, use definition of integral as area under the curve.

**Method Two:** Using Trigonometric identities.
A Motivating Example

Find \( \int_{-1}^{1} \sqrt{1 - x^2} \, dx \) in two different ways.

**Method One:** Geometric. Since \( y = \sqrt{1 - x^2} \) is the top half of the unit circle, use definition of integral as area under the curve.

**Method Two:** Using Trigonometric identities. Make the substitution \( x = \sin(\theta) \) and use \( \cos^2(\theta) + \sin^2(\theta) = 1 \).
For integrals involving $\sqrt{a^2 - x^2}$, $\sqrt{x^2 - a^2}$ or $\sqrt{x^2 + a^2}$ where $a$ is a constant, we can often integrate by constructing a right triangle with one of these values as a side length.
For integrals involving $\sqrt{a^2 - x^2}$, $\sqrt{x^2 - a^2}$ or $\sqrt{x^2 + a^2}$ where $a$ is a constant, we can often integrate by constructing a right triangle with one of these values as a side length.

**Method Three:** Find $\int_{-1}^{1} \sqrt{1 - x^2} \, dx$ by building the relevant right triangle and making a substitution.
For integrals involving $\sqrt{a^2 - x^2}$, $\sqrt{x^2 - a^2}$ or $\sqrt{x^2 + a^2}$ where $a$ is a constant, we can often integrate by constructing a right triangle with one of these values as a side length.

**Method Three:** Find $\int_{-1}^{1} \sqrt{1 - x^2} \, dx$ by building the relevant right triangle and making a substitution.

**Example:** Find $\int \frac{1}{x^2 \sqrt{x^2 + 9}} \, dx$

**Example:** Find $\int \frac{1}{\sqrt{x^2 - 4}} \, dx$