Math 123: Volumes

Ryan Blair

CSU Long Beach

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Outline

1. Review

2. Intro to Volumes

3. Volumes of Rotation

Area between Curves

Find the area between the following curve and the x-axis

$$y = 4 - x^2$$

by integrating with respect to x.

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1. Sketch the graphs and label roots of $4 - x^2$.
2. Draw rectangles representing the infinitesimal area
3. Integrate the infinitesimal area with respect to x to find the total area.

Volume Basics

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Exercise 3: Slicing into vertical ... shells.

Volume of a Paraboloid

Find the volume of the solid obtained by rotating the region bounded by $y = x^2$, $x = 0$ and $y = 4$ about the y-axis by

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Exercise 2: Slicing into vertical shells.

Volumes of solids of rotation

Replace all x's with y's in the following formulas to get other valid expressions for volume.

Disks:

Vol $= \int_a^b \pi(\text{radius in terms of x})^2 dx$

Shells:

Vol $= \int_a^b 2\pi(\text{radius in terms of x})(\text{height in terms of x}) dx$

Washers:

Vol $=$
$\int_a^b \pi(\text{outer radius in terms of x})^2 - \pi(\text{inner radius in terms of x})^2 dx$

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Exercise: Find the volume of the object obtained by rotating the region bounded by the lines $y = x$, $y = 1$ and $x = 0$ about the x-axis.