Math 123: First Order D.E.s and Slope Fields

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Thursday November 21, 2013
Outline

1. First Order Differential Equations

2. Slope Fields
Types of Differential equations

Definition
A differential equation is any equation involving a function, its derivatives.

Definition
If the n-th derivative is the largest derivative that appears in the differential equation, we say it is an **nth order** differential equation.

Definition
A first order differential equation $\frac{dy}{dx} = F(x, y)$ together with and **initial condition** $y(a) = b$ is an **initial value problem**
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Example: Solve the initial value problem \( y' = \frac{1}{2}y \) and \( y(0) = 2 \)
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Example: Solve the initial value problem \( y' = \frac{1}{2}y \) and \( y(0) = 2 \)
Example: Solve the initial value problem \( \frac{dy}{dx} = \frac{xe^x}{cos(y)} \) and \( y(0) = 0 \)
Slope Fields (Direction Fields)

Definition

Given a first order D.E. $\frac{dy}{dx} = F(x, y)$ a **slope field** is a function that assigns the slope $F(x, y)$ to each point in the plane.

Example:

Sketch the slope field for $y' = (y - 1)(y - 3)$.

Example:

Sketch the solution for the IVP $y' = (y - 1)(y - 3)$ and $y(0) = 2$. What about $y(0) = 0$.

Example:

Find $\lim_{x \to \infty} y(x)$ if $y(x)$ is a solution to the IVP $y' = (y - 1)(y - 3)$ and $y(0) = 0$. 
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Slope Fields Using Dfield

Here we will be using the free internet software Dfield.

**Example:** Determine the limits as $x$ goes to infinity for solutions to $y' = \left(\frac{1}{2}y(5 - y)\right)$ (A Verhulst Equation).

**Example:** Determine the initial values for which solutions to $y' = x^2 + y^2 - 4$ are **always** increasing.