Math 123: Power Series

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Outline

1. Power Series
A **Power Series** is a series and a function of the form

\[ P(x) = \sum_{k=0}^{\infty} c_k (x - a)^k = c_1 + c_2 (x - a) + c_3 (x - a)^2 + \ldots \]

where \( x \) is a variable, the \( c_i \) are constants and we say \( P(x) \) is centered at \( a \).

**For what values of \( x \) does a power series converge?**
Convergence of Power Series

A power series $\sum_{k=0}^{\infty} c_k (x - a)^k$ fits into one of the following three categories:

1. It converges for all $x$
2. It converges only at $x = a$
3. It converges for all $x$ such that $|x - a| < R$ where $R$ is some positive constant and may or may not converge at $x = a + R$ and $x = a - R$.

In the third case $R$ is called the radius of convergence.
Ratio Test

Theorem

Given a series $\sum_{i=1}^{\infty} a_i$. If

$$\lim_{i \to \infty} \left| \frac{a_{i+1}}{a_i} \right| = L,$$

then

1. If $L < 1$, the series converges absolutely.
2. If $L = 1$, the test is inconclusive.
3. If $L > 1$, the series diverges.
How to find the radius of convergence.

Using the ratio test we see that $\sum_{k=0}^{\infty} c_k(x - a)^k$ converges if

$$\lim_{k \to \infty} \left| \frac{c_{k+1}(x - a)}{c_k} \right| < 1$$

So, we get

$$R = \lim_{k \to \infty} \left| \frac{c_k}{c_{k+1}} \right|$$
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Find the radius of convergence for the power series \( \sum_{k=1}^{\infty} \frac{(3x)^k}{k5^k} \).
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If the radius of convergence of a power series is \( R \), find the radius of convergence of its derivative.
Given a power series $\sum_{k=0}^{\infty} c_k (x - a)^k$ with radius of convergence $R$, the interval of convergence is one of the following where we include endpoints if the series is convergent at those points.

$$(a - R, a + R), [a - R, a + R), (a - R, a + R], [a - R, a + R]$$

Find the interval of convergence of $\sum_{k=1}^{\infty} \frac{(x)^k}{k}$. 