1. Homework 4

Due: In Lecture 9-28

Problem 1. Show that if $M$ is a $k$-dimensional differentiable manifold with boundary in $\mathbb{R}^n$, then $\partial M$ is a $(k-1)$-dimensional manifold (without boundary) in $\mathbb{R}^n$.

Problem 2. Identify the set of real $2 \times 2$ matrices with $\mathbb{R}^4$.
   (a) Show that the subset $M$ of matrices of rank 1 is a 3-dimensional differentiable manifold in $\mathbb{R}^4$.
   (b) Show that the set of $2 \times 2$ matrices of determinant 1 is a 3-dimensional differentiable submanifold of $\mathbb{R}^4$.

Problem 3 Show that $M_x$ consists of the tangent vectors to smooth curves in $M$ passing through $x$.

Problem 4 (a) Find a basis for the tangent space to the unit 2-sphere $S^2$ in $\mathbb{R}^3$ at the point $p = (a, b, c)$.
   (b) What is the tangent space to the hyperboloid in $\mathbb{R}^3$ defined by the equation $x^2 + y^2 - z^2 = a^2$ at the point $(a, 0, 0)$?

Problem 5 The orthogonal group $O(n)$ consists of all $n \times n$ matrices $A$ such that $AA^T = I$. Identify the set $M(n)$ of all $n \times n$ matrices with Euclidean space of dimension $n^2$, and then show that the orthogonal group $O(n)$ is a differentiable submanifold. What is its dimension?