Cost-Volume-Profit Analysis

Understanding the relationship between a firm's costs, profits and its volume levels is very important for strategic planning. When you are considering undertaking a new project, you will probably ask yourself, “How many units do I have to produce and sell in order to Break Even?” The feasibility of obtaining the level of production and sales indicated by that answer is very important in deciding whether or not to move forward on the project in question.

Similarly, before undertaking a new project, you have to assure yourself that you can generate sufficient profits in order to meet the profit targets set by your firm. Thus, you might ask yourself, “How many units do I have to sell in order to produce a target income?” You could also ask, “If I increase my sales volume by 50%, what will be the impact on my profits?” This area is called Cost-Volume-Profit (CVP) Analysis.

In this discussion we will assume that the following variables have the meanings given below:

\[ P = \text{Selling Price Per Unit} \]
\[ x = \text{Units Produced and Sold} \]
\[ V = \text{Variable Cost Per Unit} \]
\[ F = \text{Total Fixed Costs} \]
\[ Op = \text{Operating Profits (Before Tax Profits)} \]
\[ t = \text{Tax rate} \]

Break-Even Point

Your Sales Revenue is equal to the number of units sold times the price you get for each unit sold:

\[ \text{Sales Revenue} = Px \]

Assume that you have a linear cost function, and your total costs equal the sum of your Variable Costs and Fixed Costs:

\[ \text{Total Costs} = Vx + F \]

When you Break Even, your Sales Revenue minus your Total Costs are zero:

\[ \text{Sales Revenue} – \text{Total Costs} = 0 \]
This is the “Operating Income Approach” described in your book. If you move your Total Costs to the other side of the equation, you see that your Sales Revenue equals your Total Costs when you Break Even:

\[ \text{Sales Revenue} = \text{Total Costs} \]

Now, solve for the number of units produced and sold (\(x\)) that satisfies this relationship:

\[
\begin{align*}
\text{Revenue} &= \text{Total Costs} \\
P_x &= V_x + F \\
P_x - V_x &= F \\
x(P - V) &= F \\
x &= \frac{F}{P - V} \quad \text{(FORMULA "A")}
\end{align*}
\]

Formula "A" is the “Contribution Margin Approach” that is described in your book. You can see that both approaches are related and produce the same result.

**Break-Even Example**

Assume Bullock Net Co. is an Internet Service Provider. Bullock offers its customers various products and services related to the Internet. Bullock is considering selling router packages for its DSL customers. For this project, Bullock would have the following costs, revenues and tax rates:

\[
\begin{align*}
P &= $200 \\
V &= $120 \\
F &= $2,000 \\
\text{Tax Rate (t)} &= 40\%
\end{align*}
\]

Using Formula “A”, we can compute the Break-Even Point in units:

\[
x = \frac{F}{(P - V)} = \frac{2,000}{80} = 25 \text{ units}
\]

Sometimes, you see the \((P-V)\) replaced by the term "Contribution Margin Per Unit" (CMU):

\[
x = \frac{F}{\text{CMU}} \quad \text{(FORMULA "A")}
\]

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This is way that your textbook presents Formula “A”.

This makes sense if you think about it. Every time that you sell a unit, you earn the Contribution Margin per unit. The Contribution Margin per unit is the portion of the Sales Price that is left after paying the Variable Cost per unit. It is available to pay the Fixed Costs. If every time you sell a unit you earn $80 to help pay your Fixed Costs of $2,000, how many units do you need to sell in order to pay off the $2,000 completely?

\[ x = \frac{2,000}{80} = 25 \]

### Break-Even Point In Sales Dollars

Taking Formula "A," you can multiply both sides of the equation by P:

\[ x = \frac{F}{(P-V)} \]

\[ Px = \frac{F \times P}{(P-V)} \]

Recall what you do when you have a fraction in the denominator of a fraction:

\[ \frac{a}{b/c} = \frac{(a) \times (c)}{b} \]

This works backwards as well:

\[ \frac{(a) \times (c)}{b} = \frac{a}{b/c} \]

We can rewrite this equation:

\[ Px = \frac{F}{(P-V)} \quad \text{(FORMULA "B")} \]

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Formula “B” gives you the Sales Revenue that you need in order to Break Even. The
Denominator \( [(P-V)/P] \) is referred to as the “Contribution Margin Ratio”. It tells you, what
percentage of every dollar of Sales Revenue goes to help pay off the Fixed Costs. You
can see this if you break up the Contribution Margin Ratio:

\[
\frac{(P-V)}{P} = \frac{P}{P} - \frac{V}{P} = 1 - \frac{V}{P}
\]

\( V/P \) gives you the percentage of the Sales Price that goes to pay off the Variable Costs
(the Variable Cost Ratio or Variable Margin). Thus, one minus the Variable Cost Ratio
gives you the percentage of the Sales Price that is available to help pay the Fixed
Costs.

Sometimes Formula B is rewritten by replacing \( [(P-V)/P] \) with the Contribution Margin
Ratio (CMR):

\[
P_x = \frac{F}{CMR} \quad \text{(FORMULA "B")}
\]

This is the way Formula B is presented in your book

**Break-Even Point In Sales Dollars Examples**

Let us continue using the Bullock example. Using Formula “B”, we can compute the
Break-Even Point in Sales Revenue:

\[
P_x = \frac{2,000}{(200 - 120)} \quad \sharp\quad 200
\]

\[
P_x = \frac{2,000}{.40}
\]

\[
P_x = $5,000
\]

So, what is the big deal? We already knew that Bullock needed to sell 25 units to Break
Even by using Formula “A”. We also know that each unit sells for $200. We therefore
know that selling the 25 units will produce Sales Revenue of $5,000. Why do we need
a separate formula? We have the two formulas because sometimes you might not have
enough information to use Formula “A”, but you will have enough information to use
Formula “B”.

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For example, Cuba Radio Co produces portable sports radios. It has released the following Variable Costing Income Statement. This is the only financial information that we have regarding the Cuba’s operations:

<table>
<thead>
<tr>
<th>Description</th>
<th>Amount</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sales Revenue</td>
<td>$100,000</td>
</tr>
<tr>
<td>Less Variable Costs</td>
<td>-$30,000</td>
</tr>
<tr>
<td>Contribution Margin</td>
<td>$70,000</td>
</tr>
<tr>
<td>Less Fixed Costs</td>
<td>-$50,000</td>
</tr>
<tr>
<td>Operating Profit</td>
<td>$20,000</td>
</tr>
</tbody>
</table>

What is the Break-Even point for Cuba? We do not know the number of units that Cuba sells in a year. We do not know the Price or the Variable Cost per unit. For all we know, Cuba sells one radio for $100,000 each (or 100,000 radios for $1 each). So, we cannot use Formula “A”. Although you do not know the price or the Variable Cost per unit, you are still able to calculate the Contribution Margin Ratio.

\[
\frac{\text{Contribution Margin}}{\text{Sales Revenue}} = \frac{\text{P}_x - \text{V}_x}{\text{P}_x} = \frac{\text{(P-V)}_x}{\text{P}_x} = \frac{\text{(P-V)}}{\text{P}}
\]

Thus, we can use Formula “B”. The Contribution Margin Ratio is .70 (70,000/100,000), and the Break-Even Point in Sales Revenue is:

\[\text{P}_x = \frac{\text{F}}{\text{CMR}} = \frac{50,000}{.70} = $71,428.57\]

Keep in mind that the reason that Cuba’s Sales Revenue is lower than it was before is because Cuba sold fewer units. Cuba’s price and Variable Cost per unit remained unchanged. Let’s check if Cuba Breaks Even at this Sales Revenue figure:

<table>
<thead>
<tr>
<th>Description</th>
<th>Amount</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sales Revenue</td>
<td>$71,428.57</td>
</tr>
<tr>
<td>Less Variable Costs (30%)</td>
<td>-$21,428.57</td>
</tr>
<tr>
<td>Contribution Margin</td>
<td>$50,000</td>
</tr>
<tr>
<td>Less Fixed Costs</td>
<td>-$50,000</td>
</tr>
<tr>
<td>Operating Profit</td>
<td>$0</td>
</tr>
</tbody>
</table>

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Profit Targets

You can use this same analysis to figure out how many units you need to sell in order to generate a target before-tax profit (Operating Profits).

Operating Profits are determined as follows:

\[
\text{Operating Profits} = \text{Revenue} - \text{Costs} \\
\text{Op} = P_x - V_x - F
\]

If you move the costs to the other side of the equation, you end up with:

\[
P_x = V_x + F + \text{Op}
\]

If you solve for \(x\), you will see how many units you need to produce and sell in order to generate your target Operating Profits:

\[
x = \frac{(F + \text{Op})}{(P - V)} \quad \text{Modified Formula “A”}
\]

Or \(x = \frac{(F + \text{Op})}{\text{CMU}}\)

As was true with Formula “B”, we can multiply both sides of Modified Formula “A” by price to produce the formula that gives the Sales Revenue that is necessary to produce the target Operating Profits:

\[
x = \frac{(F + \text{Op})}{(P - V)} \\
P_x = \frac{(F + \text{Op})P}{(P - V)} \quad \text{Modified Formula “B”}
\]

Or \(P_x = \frac{(F + \text{Op})}{\text{CMR}}\)

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### Profit Target Example

Assume that Bullock Net Co. has established a target Operating Profits figure of $40,000. Using Modified Formula “A”, you can determine the number of units that Bullock will need to sell in order to generate this target:

\[
x = \frac{(2,000 + 40,000)}{(200 - 120)}
\]

\[
x = \frac{42,000}{80}
\]

\[
x = 525 \text{ units}
\]

If you think about it, it makes sense to add the Fixed Costs and the Target Operating Profits together and then divide by the Contribution Margin. If you make $80 every time you sell a unit, then you have to sell 25 units to Break Even (2,000/80). After you Break Even, you make $80 of profits every time that you sell a unit, and you have to sell 500 units in order to generate Operating Profits of $40,000 (40,000/80).

Using Modified Formula “B”, you can determine the Sales Revenue that Bullock will need in order to generate Operating Profits of $40,000:

\[
x = \frac{(2,000 + 40,000)}{(200 - 120)}
\]

\[
x = \frac{42,000}{.4}
\]

\[
x = $105,000
\]

### After-Tax Profit Targets

The Operating Profits to which we have been referring do not include tax expense. Once you subtract your tax expense from your Operating Profits, you have your Net Income.

If you want to know how many units that you need to produce and sell in order to generate a target Net Income (or after-tax profit), just convert the after-tax number into a before-tax number. You can then substitute the before-tax profit figure in the above formulas.
For example, if you are told that you want to generate a Net Income (after-tax) of $50,000 and your tax rate is 40%, then you can convert the $50,000 after-tax, Net Income into the before-tax, Operating Profits that you need in order to produce Net Income of $50,000:

\[
\text{Operating Profits - Taxes} = \text{Net Income} \\
\text{Op} - .4 (\text{Op}) = 50,000 \\
.6 (\text{Op}) = 50,000 \\
\text{Op} = 50,000/.6 \\
\text{Op} = 83,334
\]

You can check this:

| Operating Profits: | $83,334 |
| Taxes (40%): | -33,334 |
| Net Income: | $50,000 |

**Targeted Income As A Percent Of Sales Revenue**

What if you are given a before tax target income, which is equal to a percentage of Sales Revenue? Just plug a formula for the target (e.g., .1Px for 10% of Sales Revenue) into the formula in place of "Op" (instead of a dollar figure). For example, assume that Bullock Net Co. desires a target Operating Profits that are equal to 10% of its Sales Revenue:

\[
P_x = (2,000 + .1P_x) / [(200 - 120)/200] \\
P_x = 2,000 + .1P_x / .40 \\
.4P_x = 2,000 + .1P_x \\
.3P_x = 2,000 \\
P_x = 2,000/.3 \\
P_x = $6,667
\]

**Multiple-Product Analysis**

What if you are interested in performing a CVP analysis, but you have more than one product? You can perform this analysis in the same manner as we described above if you assume that your sales mix is fixed.

You use the same formulas that are described above, but you substitute a composite Contribution Margin (for the entire product line) in place of the Contribution Margin for one product that we used above. When using a version of Formula “B”, you need to calculate the Contribution Margin Ratio for all of your products.
Assume that you have a Company that sells two models, Good and Better:

<table>
<thead>
<tr>
<th></th>
<th>Good</th>
<th>Better</th>
</tr>
</thead>
<tbody>
<tr>
<td>Price:</td>
<td>$60</td>
<td>$80</td>
</tr>
<tr>
<td>Variable Costs:</td>
<td>44</td>
<td>56</td>
</tr>
<tr>
<td>Units Sold:</td>
<td>1800</td>
<td>600</td>
</tr>
</tbody>
</table>

Total fixed expenses are $39,600

Construct an income statement for the company:

- Sales: \((1800 \times 60 = 108K) + (600 \times 80 = 48K)\) = $156,000
- Variable Costs: \((1800 \times 44 = 79.2K) + (600 \times 56 = 33.6K)\) = $112,800
- Contribution Margin: 43,200
- Fixed Costs: $39,600
- Operating Profits: -$8,400

The Contribution Margin Ratio for the Company is 27.6923% \((43,200/156,000)\)

You can also get the Contribution Margin Ratio for the Company by calculating the individual Contribution Margin Ratios for each product:

- Colonial
  - Price: $60
  - Variable Costs: 44
  - Contribution Margin: $16
  - Contribution Margin Ratio: 26.67%

- Early American
  - Price: $80
  - Variable Costs: 56
  - Contribution Margin: $24
  - Contribution Margin Ratio: 30%

What you have to remember, however, is that the product mix (when calculating Contribution Margin Ratios) is based on relative sales revenue of the product (not the relative units sold):

<table>
<thead>
<tr>
<th></th>
<th>Colonial</th>
<th>Early American</th>
</tr>
</thead>
<tbody>
<tr>
<td>Product Sales Revenue:</td>
<td>$108K</td>
<td>$48K</td>
</tr>
<tr>
<td>(\div)Total Sales Revenue:</td>
<td>(\div) 156K</td>
<td>(\div) 156K</td>
</tr>
<tr>
<td>Product Mix:</td>
<td>69.23%</td>
<td>30.77%</td>
</tr>
</tbody>
</table>

Weighted Average Contribution Margin Ratio:

\[ .6923(0.2667) + .3077(0.30) = .18463641 + .09231 = .2769461 \]

The difference between the two Contribution Margin Ratios for the Company is due to rounding.

Using Formula B, you get the Break Even point in Dollars

\[ PX = F/CMR = \frac{39,600}{.276923} = 143,000 \]

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If you use a version of Formula “A”, you have to come up with composite Contribution Margin per Unit that represents the entire product line. In constructing the composite Contribution Margin Per Unit, the sales mix is based on the relative number of **UNITS** sold of each product (not the relative dollar sales revenue).

There are two methods that are commonly used to develop the needed composite variables: (i) Basket (Package) Method, and (ii) Weighted Average Contribution Margin Method.

A major competitor of Carmen’s Banana Business, Inc. is Woody’s Bananas, Ltd. Competition between Woody and Carmen has become so fierce on the Banana front that Woody has been suspected of engaging in guerilla tactics. (This suspicion could just be based on the CEO’s strange fashion statements.) Unlike Carmen, who specializes in various banana products, Woody only sells raw fruit. Woody’s main emphasis is Bananas, but it also sells Oranges. Woody sells 3 Bananas for each Orange that it sells (75% vs. 25%).

<table>
<thead>
<tr>
<th></th>
<th>Bananas</th>
<th>Oranges</th>
</tr>
</thead>
<tbody>
<tr>
<td>Price</td>
<td>$2</td>
<td>$4</td>
</tr>
<tr>
<td>VC</td>
<td>$1</td>
<td>$2</td>
</tr>
<tr>
<td>CM</td>
<td>$1</td>
<td>$2</td>
</tr>
</tbody>
</table>

Common Fixed Costs: $2,000

With the Basket Method, you create a Basket that reflects Woody’s sales mix (75%:25%). Assume that each basket contains 3 Bananas and 1 Orange. What is the Contribution Margin of that Basket?

\[
\begin{align*}
\text{CM}_\text{basket} &= 3 \cdot \text{CM}_\text{banana} + 1 \cdot \text{CM}_\text{orange} \\
\text{CM}_\text{basket} &= 3(1) + 1(2) \\
\text{CM}_\text{basket} &= 3 + 2 \\
\text{CM}_\text{basket} &= 5
\end{align*}
\]

Now, you plug the Contribution Margin for the Basket into the Formula that you wish to use. The Break-Even point in Baskets is:

\[
\text{Baskets} = \frac{\text{F}}{\text{CM}_\text{basket}}
\]

\[
\begin{align*}
\text{Baskets} &= \frac{2,000}{5} \\
\text{Baskets} &= 400 \text{ Baskets}
\end{align*}
\]

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You now describe the units of fruit contained in the Baskets:

**Bananas:** There are 3 Bananas in every Basket, so we need to sell $3 \times 400$ Baskets = 1200 Bananas in order to Break-Even.

**Oranges:** There is one Orange in every Basket, so we need to sell 400 Oranges in order to Break-Even.

With the Weighted Average Contribution Margin Method, we calculate the Weighted Average Contribution Margin for one unit of fruit, using the given sales mix.

\[
CM_{wa} = .75 \times CM_{banana} + .25 \times CM_{orange}
\]

\[
CM_{wa} = .75 \times (1) + .25 \times (2)
\]

\[
CM_{wa} = .75 + .5 = $1.25
\]

The Break-Even Point in units is:

\[
x = \frac{F}{CM_{wa}}
\]

\[
x = \frac{2,000}{1.25}
\]

\[
x = 1600 \text{ units}
\]

Since we know that the total units of fruit sold should be 1600, and we know the sales mix is 75%:25%:

**Bananas:** .75 \times (1600) = 1200 Bananas

**Oranges:** .25 \times (1600) = 400 Oranges

---

**Margin of Safety**

The "Margin Of Safety" is the amount of sales (in dollars or units) by which the actual sales of the company exceeds the Break-Even Point. We know that Bullock's Break-Even Point is 25 units or $5,000. If Bullock really sells 40 units (Sales Revenue of $8,000), then its Margin Of Safety is 15 units (40-25) or $3,000 ($8,000 - $5,000).
**Operating Leverage**

If you take the total Contribution Margin and divide it by the Operating Profits, this gives you the Operating Leverage (or degree of Operating Leverage).

For example, if Bullock Net Co had actual sales of 40 units, its Operating Profits would be calculated as follows:

- **Revenue**: $8,000  
  - (200 x 40)
- **Variable Costs**: -$4,800  
  - (120 x 40)
- **Contribution Margin**: $3,200
- **Fixed Costs**: -$2,000
- **Operating Profits**: $1,200

The Operating Leverage is calculated as follows:

\[
\frac{\text{Contribution Margin}}{\text{Operating Profits}} = \frac{3,200}{1,200} = 2.67
\]

The Operating Leverage of 2.67 indicates that if Bullock can increase its sales by 50%, then its Operating Profits will increase by 2.67 x 50% or 133%. Thus, the Operating Profits of $1,200 will increase by $1,600 (1.33 x $1,200) to $2,800. This calculation assumes that the Price, Variable Cost per Unit, and the Fixed Costs do not change. You are assuming that the increase in sales is caused by a 50% increase in the number of units sold (x):

<table>
<thead>
<tr>
<th></th>
<th>OLD</th>
<th>NEW</th>
</tr>
</thead>
<tbody>
<tr>
<td>Revenue:</td>
<td>$8,000</td>
<td>$12,000</td>
</tr>
<tr>
<td></td>
<td>(200x40)</td>
<td>(200 x 60)</td>
</tr>
<tr>
<td>Variable Costs:</td>
<td>-$4,800</td>
<td>-$7,200</td>
</tr>
<tr>
<td></td>
<td>(120x40)</td>
<td>(120 x 60)</td>
</tr>
<tr>
<td>Contribution Margin:</td>
<td>$3,200</td>
<td>$4,800</td>
</tr>
<tr>
<td></td>
<td>(80 x 60)</td>
<td></td>
</tr>
<tr>
<td>Fixed Costs:</td>
<td>-$2,000</td>
<td>-$2,000</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Operating Profit:</td>
<td>$1,200</td>
<td>$2,800</td>
</tr>
</tbody>
</table>

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