**Mathematics Education Comprehensive Exams**

**Sample Questions**

**COMP EXAM #1 (Core–MTED 511 & 512)**

**For each question below you will provide (a) an outline of your response, and (b) a final response that is at least one full page in length.**

1. Choose and discuss one learning theorist that subscribed to a hierarchical view of learning (e.g., Van Hiele, Piaget, Bruner).
2. Over the past 65 years (1950–2015) of mathematics education, what have been the “big themes” within our national movement? Provide a historical sketch that addresses issues of curriculum, assessment, learning theories, and pedagogical approaches. Be sure to include specific dates/events/documents as part of your sketch.
3. What do national documents (e.g., NCTM Principles and Standards, Common Core State Standards for Mathematics) suggest about our intended national curriculum? What do research and assessments (e.g., TIMMS and PISA studies, NAEP) suggest about how this intended curriculum is enacted? How does our national curriculum compare to that of other countries? Relate your analyses to both the unique characteristics of the American educational system and international contexts.
4. In which ways does the Common Core State Standards potentially improve the quality of the school mathematics curriculum in the U.S.? What principles and main strategies for both instruction **and** assessment are essential for the Common Core Standards to be successfully implemented across different school settings? Be specific and accurate when you make references to the Common Core State Standards.

**COMP EXAM #2 (Electives–MTED 540, 550, 560, 570/590, & 580)**

MTED 540 (Dr. Li’s course):

1. What role does Algebraic Thinking play in practical applications outside of school? Response should be 2-4 paragraphs in length.



* 1. Construct the following graph for g(x), either by hand or utilizing a graphing calculator,

and provide a sketch of the graph.

1. Provide an equation for the parent function for g(x) and explain the translations you would make to arrive at g(x).
2. Find the equations for the vertical and horizontal asymptotes of g(x), and explain how you found each one of these equations.
3. Provide 2 examples of aspects of Algebraic Thinking involved in solving this problem.
4. (a) Describe three different strategies for solving a two-step equation such as 3*x* + 7 = 25.

(b) For each of the strategies, explain

* what fundamental mathematical concepts or properties are involved, and
* what misconceptions, mistakes, or difficulties you expect to see from Algebra 1 students when they learn or use this strategy.

MTED 540 (Dr. Chesler’s course):

1. One of the Common Core State Standards for Algebra in the “Reasoning with Equations & Inequalities” domain is the following:

A.REI #11. Explain why the x-coordinates of the points where the graphs of the equations y = f(x) and y = g(x) intersect are the solutions of the equation f(x) = g(x).

Explain it. In your explanation, make reference to a definition of functions (which you provide). Be sure that your explanation explicitly makes connections to both graphical and symbolic representations of functions. Illustrate your explanation with an example in which you clearly define what f(x) is and what g(x) is.

2. By making reference to mathematics education literature about the conceptions of school algebra, describe the conceptions of school algebra which are embraced in (1) the *Common Core State Standards for Mathematics*, and (2) the National Council of Teachers of Mathematics’ *Principles and Standards for School Mathematics.* For each, your answer should include (but not be limited to) answers to the following:

* What are considered to be the essential objects of algebra?
* What types of mathematical tasks/problems are commonly associated with the conception?

3. Consider the polynomial expression $4x^{2}+12x+8$. We can rewrite this as $\left(2x\right)^{2}+6(2x)+8$ and think about it as a quadratic in $2x$ and now factoring should be, more or less, as easy as factoring $z^{2}+6z+8$. You’ve demonstrated this method to your students. Now, suppose a student needs to factor $6x^{2}+11x-10$. The student says, ``If we multiply it by 6, then we can think about it as a quadratic in $6x$ and use the same method.'' The student has a great idea, though she has not provided enough information about her thinking and may not have thought through all the details (e.g., a detail to think through is that $6x^{2}+11x-10$ and $6(6x^{2}+11x-10)$ are not equivalent expressions}.

* 1. Use the student's method to correctly factor $6x^{2}+11x-10$. Don’t introduce any new variables.
	2. Create your own non-monic quadratic trinomial expression and factor it using the student's method. No need for explanation. Just clearly show your work.
	3. Describe in words and show the steps to use this strategy for a general quadratic of the form $ax^{2}+bx+c$, $a\ne 1$. Do this up until the point where you can write something like,``Now factor the expression as a quadratic in \_\_\_\_\_\_''. You don't need to describe how to factor and your answer here should be fairly short.
	4. Consider Standard for Mathematical Practice 7: SMP 7: Look for and make use of structure. Explain how the student may have looked for and made use of structure in extending her strategy.
	5. Consider Standard for Mathematical Practice 8: Look for and express regularity in repeated reasoning. Explain how this relates to parts a, b, and c of this question.

MTED 550:

1. In your opinion, how can we as teachers help students to develop critical thinking abilities in mathematics? Give a ***concrete*** example, utilizing 2-3 remediation strategies.
2. (a) Using any approach to proof, prove the following theorem: “The line segment joining

the midpoints of two sides of a triangle is parallel to the third side and its length is equal

to half of the third side.”

(b) Explain why you chose this approach.

3. Show that the adjective “included” is necessary in SAS by constructing (sketching is OK)

a counter example when the corresponding, congruent angle is not “included.” What happens if the triangle contains a right angle? Are all of the same conditions still needed?

4. State briefly the fundamental differences and similarities of Euclidean Geometry and

 Projective Geometry. Give specific examples of these similarities and differences for such

concepts and definitions of (1) parallelism, (2) concurrency, (3) collinearity, and

(4) measurement and cross-ratio.

MTED 560:

1. (a) Prove that the sum of the first n counting numbers equals  using the Principle of Mathematical Induction.
2. Explain why the process utilized in Mathematical Induction proves a theorem for all n.
3. Describe at least two common pitfalls by students using the Principle of Mathematical

Induction.

2. Find the first three derivatives of *y* = (1+2*x*)4/5 .

3. Use implicit differentiation to find the equation of the tangent line to the graph of the equation 

4. Suppose the position of an accelerating bobsled is given by **** feet . What is the speed of the bobsled when t = 1? What is the acceleration when t = 1?

5. (a) Should calculus be taught in high school? Why or why not?

 (b) Should high school calculus be as rigorous as college level calculus? Why or why not?

Support your answers based on what national data and research suggest.

6. Identify one fundamental concept in calculus and discuss the following:

 (a) Mathematically, why is this concept important?

(b) What prerequisite knowledge and skills should students have for studying this concept?

Why?

(c) What major difficulties or misconceptions may occur when students learn this concept?

What should calculus teachers do to help students overcome these difficulties and misconceptions?

MTED 570/590 (Modeling):

1. What is a mathematical model and what is mathematical modeling? What is the relationship between mathematical modeling and problem solving?
2. Outline the main stages of a mathematical modeling process. Describe the kinds of activities typically involved in each stage and teachers’ role in coordinating these activities. Use specific examples of modeling tasks.
3. What roles could mathematical modeling play in school mathematics? What strategies may teachers utilize to help students (1) improve their modeling skills and (2) learn new mathematics through modeling activities? Make sure to base your argument on the Common Core Standards as well as theoretical and empirical research.
4. Given a set of six data points A (0.8, 2), B (2.5, 4.2), C (3.5, 3.5), D (4.2, 5.3), E (5.8, 4.5), and F (7.5, 7.5),
5. fit these points with:

 a) a directly proportional trendline that goes through point F

 b) a linear trendline that goes through points A and F.

(2) For each of the above two trendlines, show its equation and calculate its R2 value. Which model better fits the original data set?

(3) Discuss how mathematics teachers can help student understand the trendline concept based on their prior knowledge and skills in algebra and geometry.

5. (1) Show that $a\_{n}=b^{n}\left(a\_{0}-\frac{m}{1-b}\right)+\frac{m}{1-b}$ is the solution to the affine discrete dynamical system $a\_{n+1}=ba\_{n}+m$ (*n* = 0, 1, 2, 3… and *b* ≠ 0)

(2) Find the equilibrium value of the above system

MTED 580:

1. **Compare and contrast the two problems below. In your response, explain what each problem is assessing. What place (if any) does each have in the secondary mathematics classroom? Why?**

**Problem 1:** Seven 100-point tests were given during the fall semester. Erika's scores on the tests were 76, 82, 82, 79, 85, 25, 83. Find Erica's mean score.

**Problem 2:** Seven 100-point tests were given during the fall semester. Erika's scores on the tests were 76, 82, 82, 79, 85, 25, 83. What grade should Erika receive for the semester?

1. The following is a problem adapted from a NAEP task:

A boy has two quarters. What is the probability that he will get one “head” and one “tails” if he flips them? Explain.

The most common answers for this question were 1/2, 1/3, and 1/4. What is the correct answer? Explain. For each of the other answers, what are some of the misconceptions that may have lead to a student providing that answer? How would you help these students correct these misconceptions? As part of your response, you may make suggestions about specific activities you would provide the student.

1. The binomial probability density function is $P\left(x\right)=\left(\begin{matrix}n\\x\end{matrix}\right)p^{x}q^{n-x}$; it represents the probability of x successes in n trials where the probability of success is p and the probability of failure is q. Give an example of a binomial experiment. Explain the formula. Be sure that you address the question: Why does the formula make sense? For example, we are multiplying three quantities... where do they come from and why does multiplying them make sense (you may refer to your example)?
2. This question is about how the fields of Probability and Statistics are connected. In the context of either confidence intervals or hypothesis testing (your choice), explain the connection between the idea of sample space in probability and variation in statistics.