Heat Transfer Analysis of Supercooled Droplets Using Finite Difference Method

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Introduction:

•Over 15% of weather-related aviation accidents is attributed to aircraft icing [1]. Aircraft icing is caused by supercooled water droplets that exist in clouds.

•The accumulated ice hinders mechanical functions of wings, reduce lift, and increase drag, all of which pose a major safety problem.

•We will explore the temperature transition and the time it takes for a suspended supercooled droplet to freeze using finite difference.

Problem [2]:

Experiment

•Cold airstream monitored by Thermocouple 2 & 3

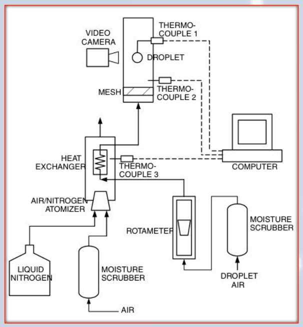
•Temperatures monitored & recorded by computer

•Liquid nitrogen & air mixed to produce cold air stream

•Moisture scrubbers remove moisture from airstream

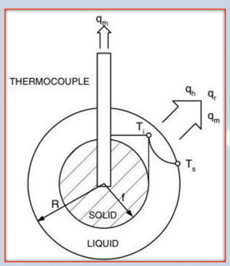
•Proper temperature were pushed to second chamber

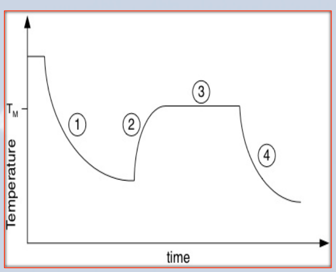
•Droplet was suspend on thermocouple 1.



Outward Freezing Schematic:

q_{th} = Thermalcouple
q_h = Heat Transfer
q_r = Thermal Radiation
q_m = Mass Transfer
R = Droplet Radius
f = Solid front





Temperature Phases

(1) Liquid Cooling

(2) Recalescence

(3) Freezing

(4) Solid Cooling

Mathematical Models [2]:

Cooling Stages (1) and (4)

Uniform Temperature Solution

$$c\rho V_d \frac{\partial T_d}{\partial t} = q_h + q_m + q_r + q_{th}$$

Internal Heat Conduction Model

$$\begin{cases} c\rho \frac{\partial T}{\partial t} = \frac{\partial}{\partial r}\left(k \frac{\partial T}{\partial r}\right) + \frac{2k}{r}\frac{\partial T}{\partial t} \\ -k\left.\frac{\partial T}{\partial r}\right|_{r=R} = q_h + q_m + q_r, \quad -k\left.\frac{\partial T}{\partial r}\right|_{r=0} = q_{th} \end{cases}$$

Recalescence Stage (2)

$$V_f = V_d \frac{c_l \rho_l}{\rho_s} \frac{(T_f - T_n)}{L_f}$$

Freezing Stage (3)

Heat Balance Model

$$L_f \rho_s \frac{\partial V_f}{\partial t} = q_h + q_m + q_r + q_{th}$$

Moving Boundary Model

$$\rho_s L_f \frac{df}{dt} = k_s \left.\frac{\partial T_s}{\partial r}\right|_{(r=i)} - k_l \left.\frac{\partial T_l}{\partial r}\right|_{(r=i)} \qquad f_i^o = R\sqrt[3]{V_f}$$

$$v = K_m(T_f - T_i) \qquad f_i^l = R\sqrt[3]{1 - V_f}$$

Finite Difference Method [3]:

Heat Equation

$$\frac{\partial T}{\partial t}(x,t) = \alpha \frac{\partial^2 T}{\partial x^2}(x,t)$$

$$\frac{\partial T}{\partial x}(0 \,\text{or}\, L, t) = f(t)$$

$$T(x,0) = g(x)$$

Forward Difference

$$\left(\frac{\partial T}{\partial t}\right)_i^n = \frac{T_i^{n+1} - T_i^n}{\Delta t}$$

Central Second Difference

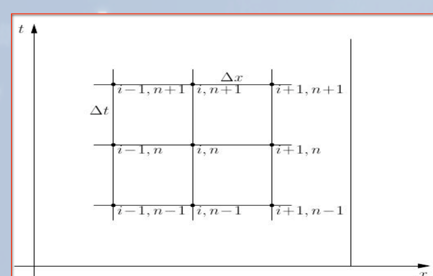
$$\left(\frac{\partial^2 T}{\partial x^2}\right)_i^n = \frac{T_{i+1}^n - 2T_i^n + T_{i-1}^n}{\Delta x^2}$$

Difference Equation

$$\frac{T_i^{n+1} - T_i^n}{\Delta t} = \alpha \frac{T_{i+1}^n - 2T_i^n + T_{i-1}^n}{\Delta x^2}$$

CFL Condition

$$\alpha \frac{\Delta t}{\Delta x^2} \le \frac{1}{2}$$



Results [2]:

Experimental data using 40 droplets were obtained. From the data, the freezing time of the droplet was estimated using the accepted definition of freezing time.

Temperature of Droplet in Solid Cooling Stage

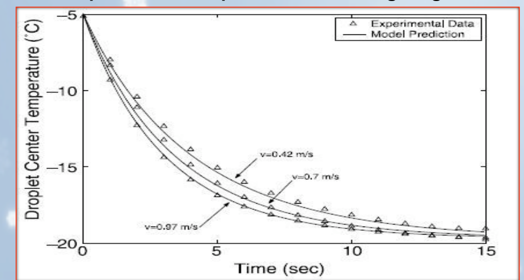


Figure 1: Using data from varying air velocity and the respective predicted model, the solid droplet takes approximately 15 cool.

Total Time and Temperature Over All Four Stages

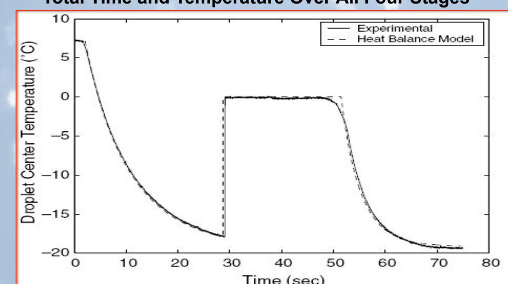


Figure 3: The droplet takes approximately 55 seconds to freeze.

Conclusions:

•With the optimal parameters, the model yield a freezing time of approximately 55 seconds.

•Using this information, Anti-freezing liquids can be produced to absorb the crystallization and prevent the freezing stage.

•For other solutes such as pollutants dissolved in droplets, a similar experiments can be conducted.

References:

[1] Lankford, T. T., Aircraft Icing: A Pilot's Guide, McGraw-Hills Companies, Inc., New York, NY, 1999.

[2] Chen, X. D., Hindmarsh, J.D., Russell, A.B., Experimental and Numerical Analysis of the Temperature Transition of a Suspended Freezing Water Droplet,

[3] John D. Anderson, Jr., Computational Fluid Dynamics, The Basics with Applications, The McGraw-Hill Companies, Inc., 1995.