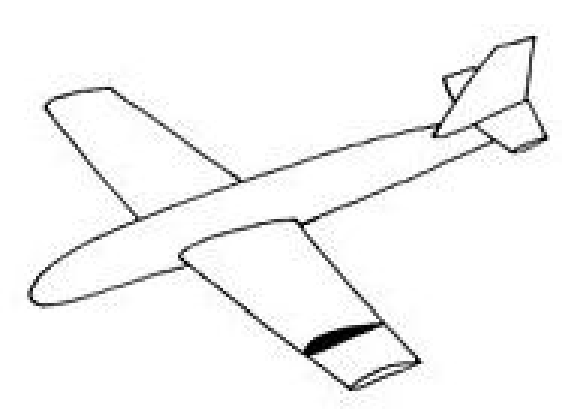
MATH 573 Advanced Scientific Computing

Analysis of an Airfoil using Cubic Splines



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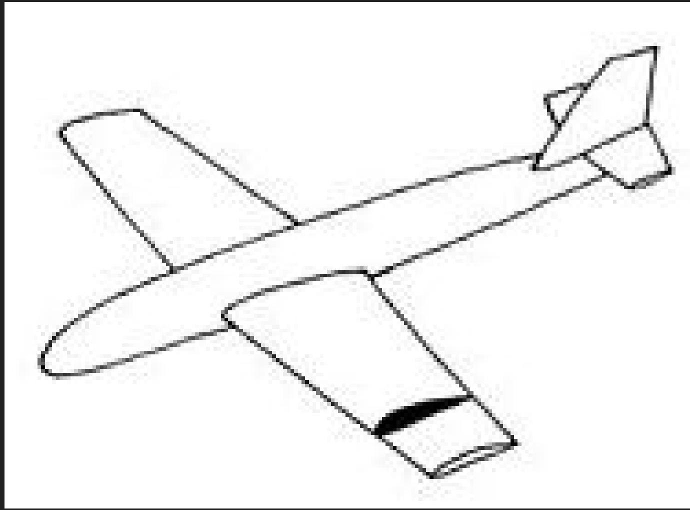
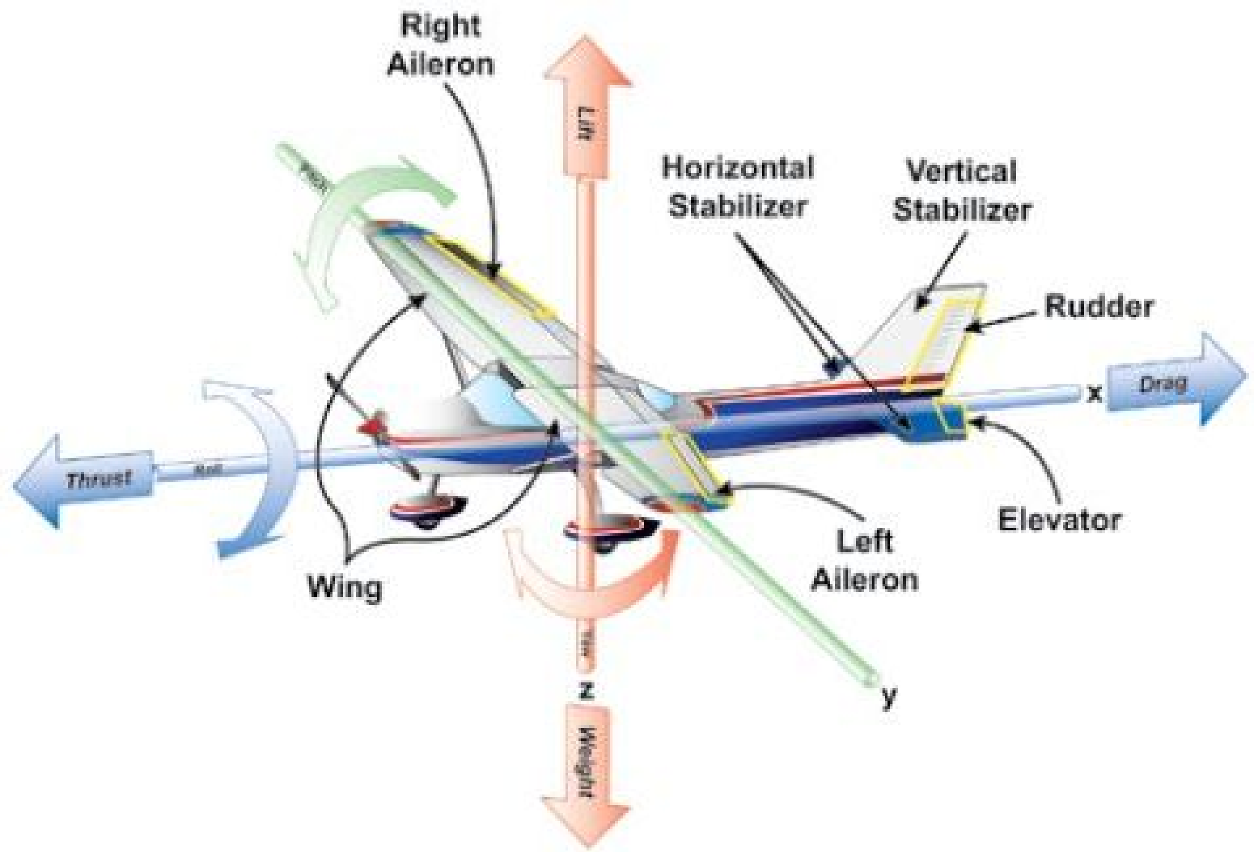
What is Airfoil?

- The cross-section of the wing, blade, or sail.

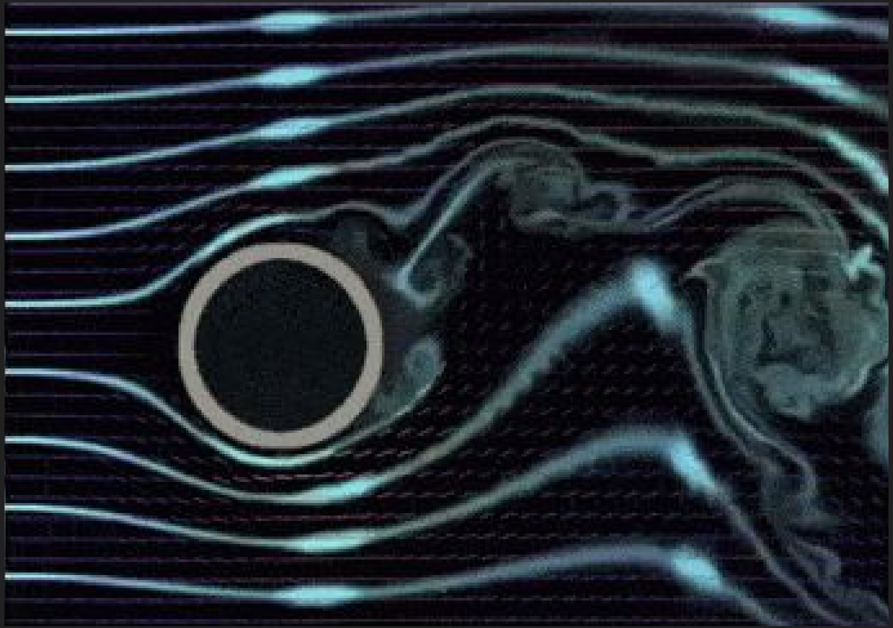


Forces Acting on an Airplane

1. Thrust
2. Weight
3. Drag
4. Lift



Airflow around a Sphere - Laminar vs Turbulent flow



Streamlines

$$Laminar\ \ flow \Rightarrow \quad Re \le 2000$$

$$Transitional\ \ flow \Rightarrow \quad 2000 < \ Re \le 4000$$

$$Turbulent\ \ flow \Rightarrow \quad Re > 4000$$

Parameters:

1. Mach number (M)
2. Reynolds number (Re)

$$M = \frac{Velocity\ of\ the\ object}{Speed\ of\ the\ sound\ (medium)} = \frac{u}{c} = \frac{u}{\sqrt{\gamma RT}}$$

$$C_L = \frac{L}{\frac{1}{2}\rho u^2 S} = f(\alpha, M, Re)$$

Calculate :

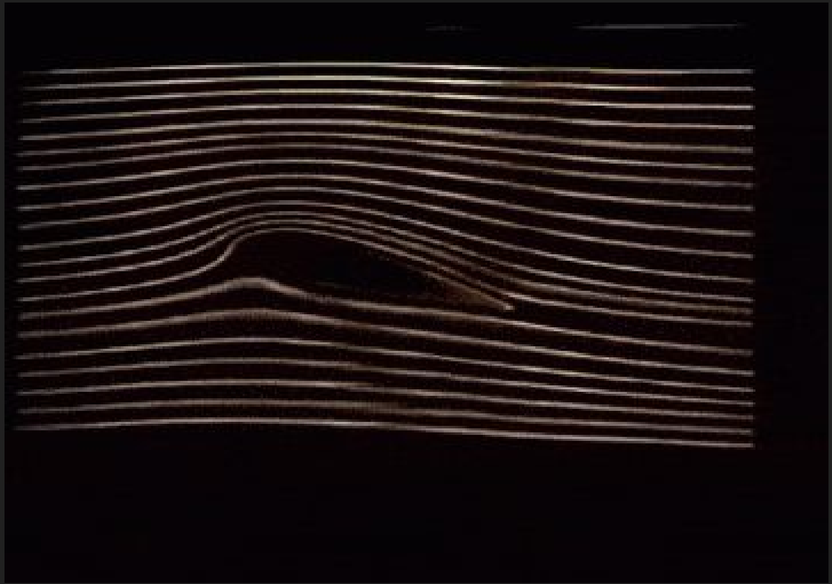
. Coefficient of Lift (C_L)
. Coefficient of Drag (C_D)

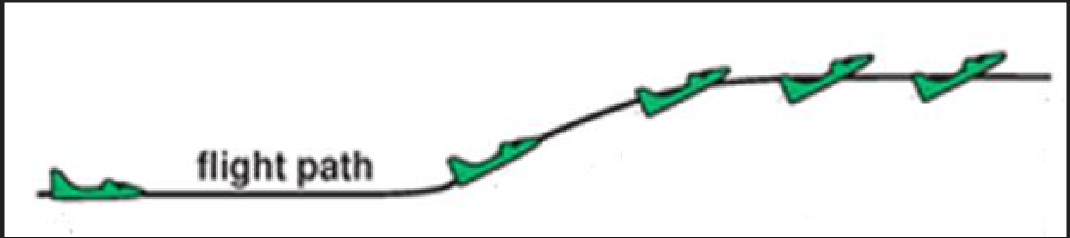
$$Re = \frac{\rho u l}{\mu} = \frac{ul}{\nu}$$

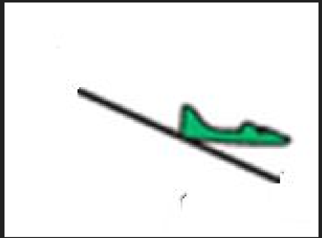
$$C_D = \frac{F_D}{\frac{1}{2}\rho u^2 A} = g(\alpha, M, Re)$$

Angle of attack - α

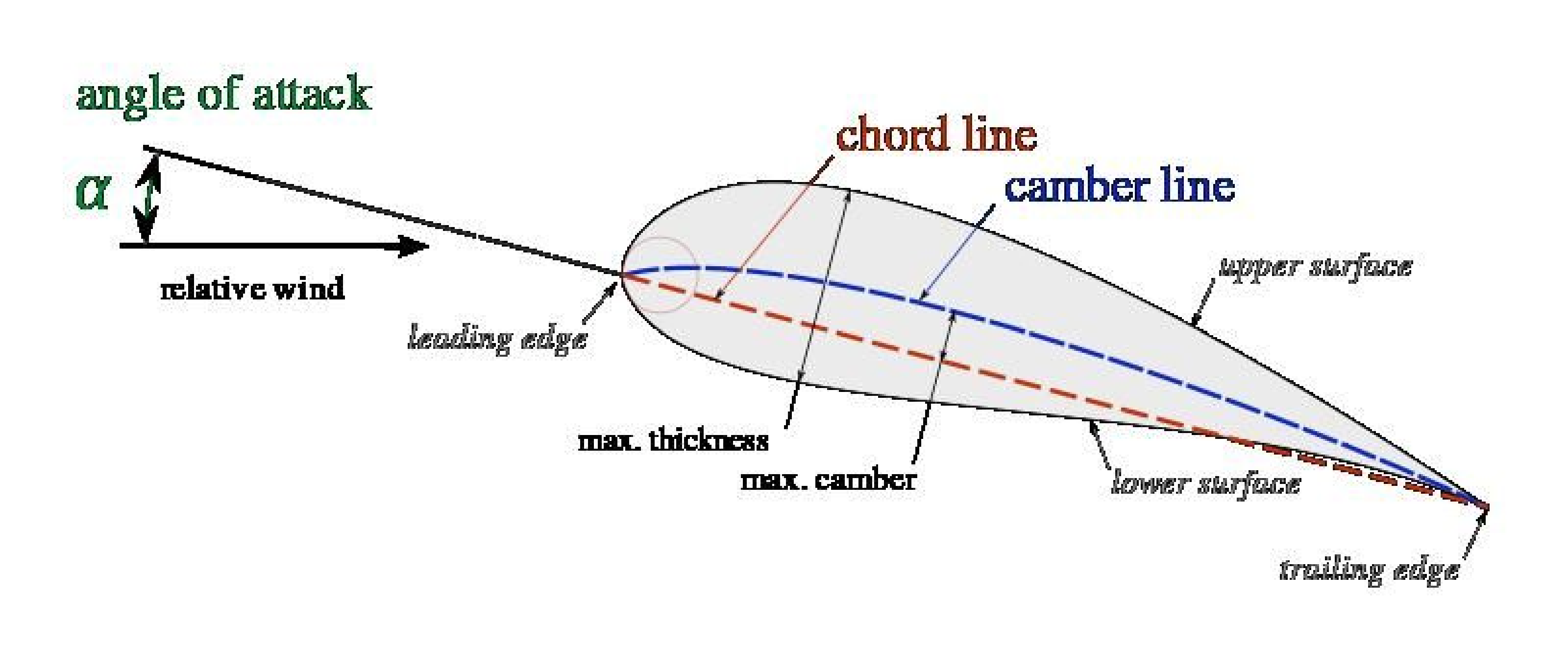
$0° < \alpha < 15°$



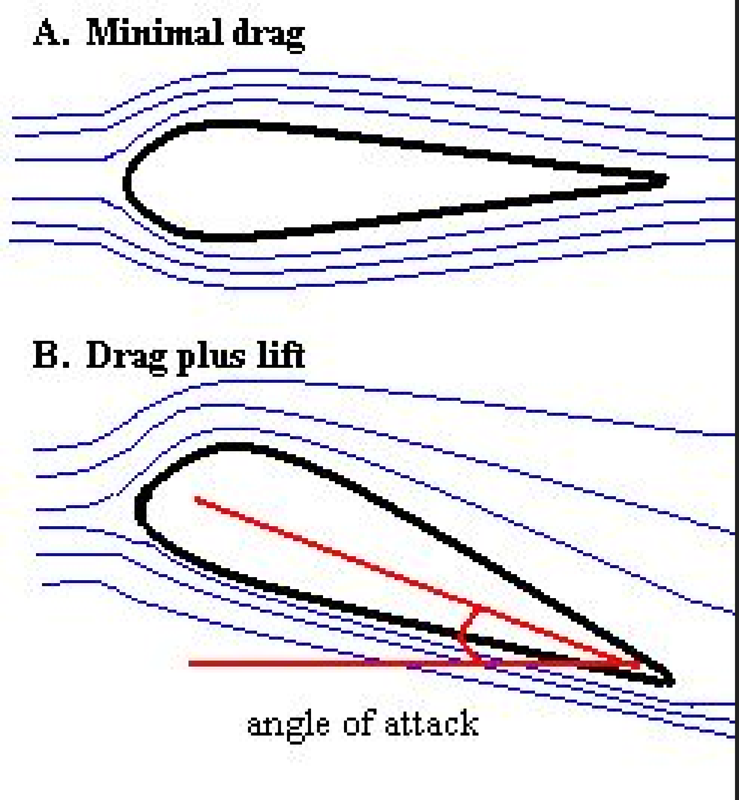




Airfoil Geometry Definitions



Cambered vs. Symmetrical Airfoils



NACA 4-Digit Airfoil Specification

NACA-MPXX

- M is the maximum camber in hundredths of a chord
- P is the position of maximum camber in tenths of a chord
- XX is the maximum thickness in hundredths of a chord

e.g. If an airfoil number is NACA-2412, the maximum camber is 0.02, or 2% of the chord, the max camber position is at 0.4, or 40% of the chord, and the max thickness is 0.12, 12% of the chord

Mean Camber Line Equations

The equation for the mean camber line is split into two parts: the region less than P and the region greater than or equal to P

Gradient of the camber line is used to calculate the upper and lower surfaces

	Front ($0 \le x < p$)	Back ($p \le x \le 1$)
Camber	$y_c = \frac{M}{P^2}\left(2Px - x^2\right)$	$y_c = \frac{M}{(1-P)^2}\left(1 - 2P + 2Px - x^2\right)$
Gradient	$\frac{dy_c}{dx} = \frac{2M}{P^2}(P - x)$	$\frac{dy_c}{dx} = \frac{2M}{(1-P)^2}(P - x)$

x ranges from zero to 1 (scaled length of chord), y_c is camber line position

Thickness Distribution Equation

$$y_t = \frac{T}{0.2}\left(a_0 x^{0.5} + a_1 x + a_2 x^2 + a_3 x^3 + a_4 x^4\right)$$

$a_0 = 0.2969$ $a_1 = -0.126$ $a_2 = -0.3516$ $a_3 = 0.2843$

$a_4 = -0.1015$ or -0.1036 for a closed trailing edge

- Constants a_0 through a_4 are for an airfoil with 20% thickness; T/0.2 adjusts for desired thickness
- Finite thickness at trailing edge; if closed edge is required, a_4 can be adjusted
- y_t is half thickness and needs to be applied to both sides of camber line

Calculating the Surfaces of an Airfoil

Using the camber line position, y_c, the gradient of the camber line, and the thickness distribution, the upper and lower surfaces of the airfoil can be calculated perpendicular to the camber line

$$\theta = \operatorname{atan}\left(\frac{dy_c}{dx}\right)$$

Upper Surface	$x_u = x_c - y_t \sin(\theta)$	$y_u = y_c + y_t \cos(\theta)$
Lower Surface	$x_l = x_c + y_t \sin(\theta)$	$y_l = y_c - y_t \cos(\theta)$

Airfoils Designed with Cubics Splines vs. Equations

Although the ability to choose desired parameters allows for some freedom in creating the airfoil shape, the shape of the airfoil is restricted to the coordinates generated by the equations

In order to allow for more freedom in generating the shape (and create less drag) we can use splines instead

Using the equations to generate points, we can then “tweak” desired coordinates to modify the shape

Airfoil Designed with Cubic Splines

Ideas prior to when this research paper was written (2007) which involve the use of multiple knots (points of reference) use about 11 total variables to generate potential airfoil

- “Curse of dimensionality”: If the design uses k locations in a one dimension space, for n dimension, it requires k^n locations
  - Barely parsimonious (cost inefficient)

Airfoil designed with cubic spline can be reduced to 6 variables.

Splines of upper half and bottom half are calculated separately with same beginning and end points.

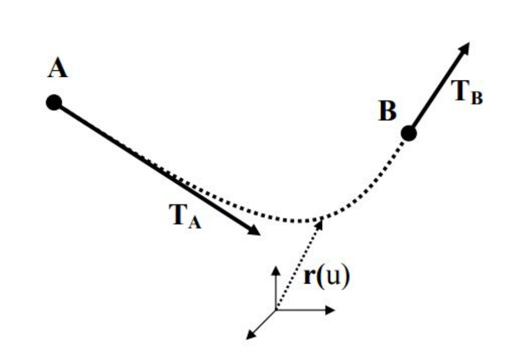
Airfoil Designed with Cubic Splines (cont.)

r(*u*) is defined as the parametric equation that generates the airfoil surface.

A, B are the two knots used for reference (leading and trailing edge of the airfoil)

T is the tangent of r(*u*) (only use when *u* = 0 and *u* = 1).

$$\mathbf{r}(u) = \sum_{i=0}^{3} \mathbf{a}_i u^i, \ u \in [0,1].$$



$$\mathbf{A} = \mathbf{a}_0$$

$$\mathbf{B} = \mathbf{a}_0 + \mathbf{a}_1 + \mathbf{a}_2 + \mathbf{a}_3$$

$$\mathbf{T}_A = \mathbf{a}_1$$

$$\mathbf{T}_B = \mathbf{a}_1 + 2\mathbf{a}_2 + 3\mathbf{a}_3.$$

Airfoil Designed with Cubic Splines (cont.)

$$\mathbf{a}_0 = \mathbf{A}$$

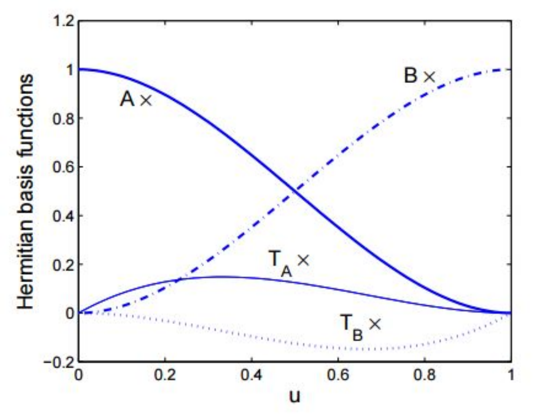
$$\mathbf{a}_1 = \mathbf{T}_A$$

$$\mathbf{a}_2 = 3[\mathbf{B} - \mathbf{A}] - 2\mathbf{T}_A - \mathbf{T}_A$$

$$\mathbf{a}_3 = 2[\mathbf{A} - \mathbf{B}] + \mathbf{T}_A + \mathbf{T}_B$$

$$\mathbf{r}(u) = \mathbf{A}(1 - 3u^2 + 2u^3) + \mathbf{B}(3u^2 - 2u^3) + \mathbf{T}_A(u - 2u^2 + u^3) + \mathbf{T}_B(-u^2 + u^3),$$

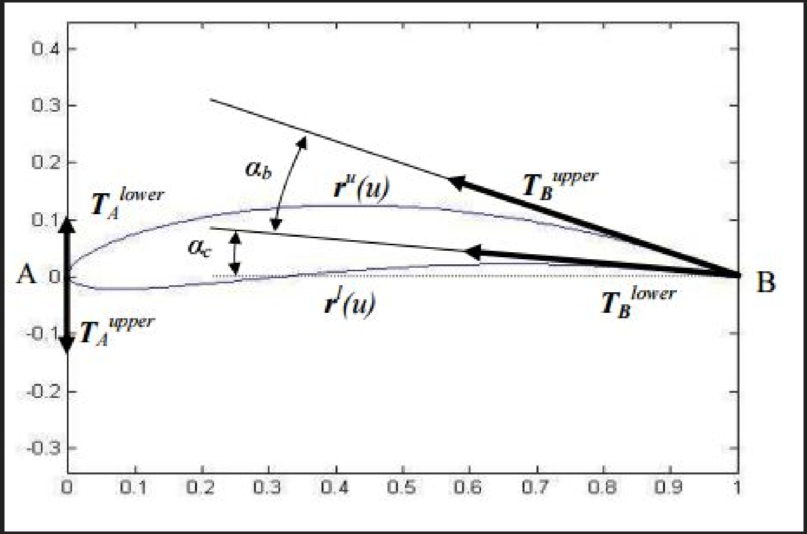
$$\mathbf{r}(u) = \begin{bmatrix} 1 & u & u^2 & u^3 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ -3 & 3 & -2 & -1 \\ 2 & -2 & 1 & 1 \end{bmatrix} \begin{bmatrix} \mathbf{A} \\ \mathbf{B} \\ \mathbf{T}_A \\ \mathbf{T}_B \end{bmatrix}$$



Airfoil Designed with Cubic Splines (cont.)

Alongside of being fewer parameters, there is also potential attributes of aerodynamic features.

- The tangents at A and B are features to represent angle of attack capability and pressure drag respectively.



Eight features for optimality

Kulfan and Bussoletti (of the American Institute of Aeronautics and Astronautics) list eight desirable features an airfoil geometry representation technique should possess:

1) smooth, well behaved and leading to realistic shapes

2) mathematically efficient

3) parsimonious

4) allow the specification of key features (e.g., leading edge radius)

Eight features (cont.)

5) allow easy control for editing

6) it should have an intuitive geometric interpretation

7) it should be systematic and consistent

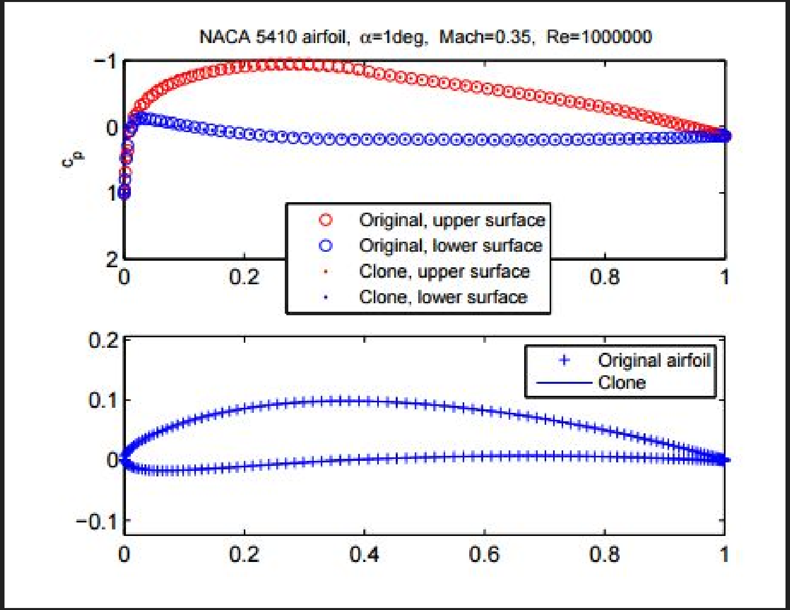
8) robust (stable)

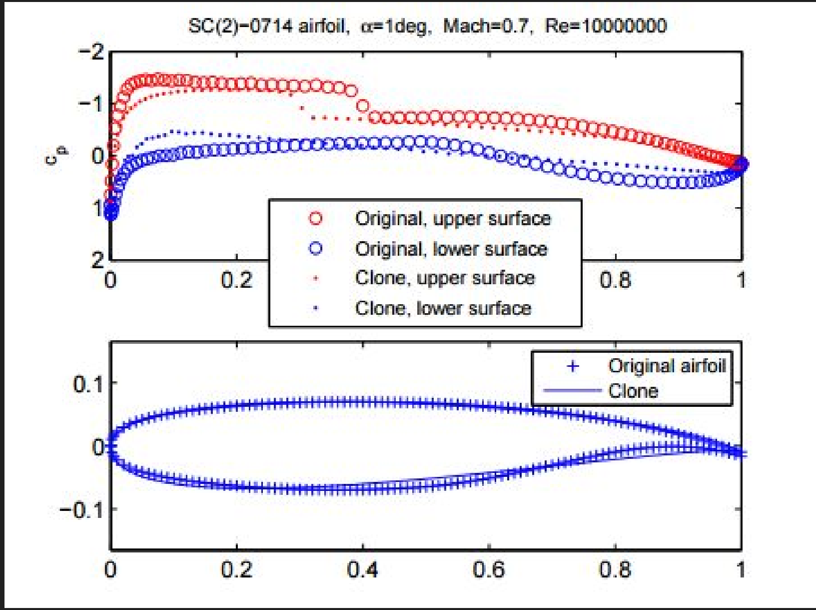
Optional:

9) Easily compatible with CAD

Reproduction of existing Airfoil

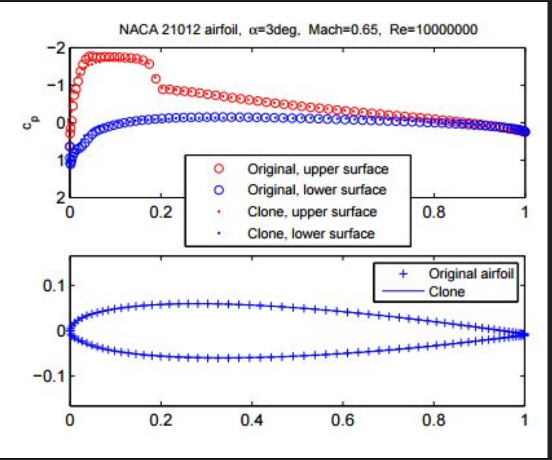
Method for verifying a functioning Cubic spline function (using two knots)

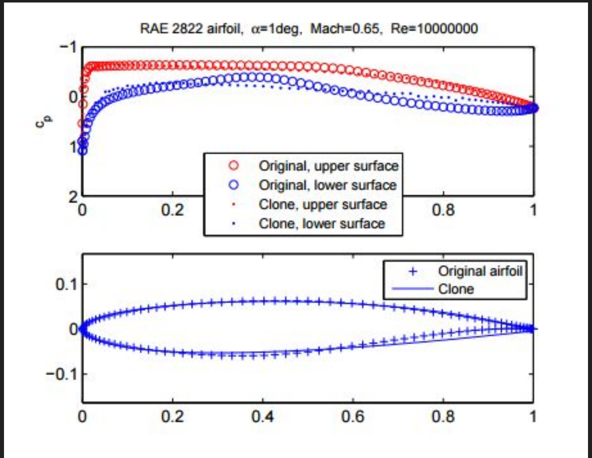


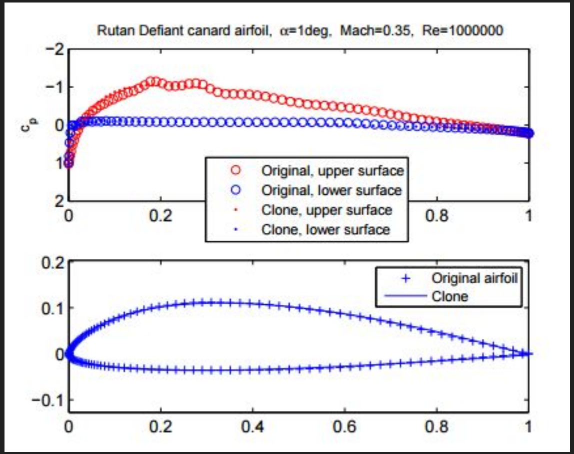


A bit off for this design - deviates in pressure profile

Generally Accurate Estimates with Six Variables



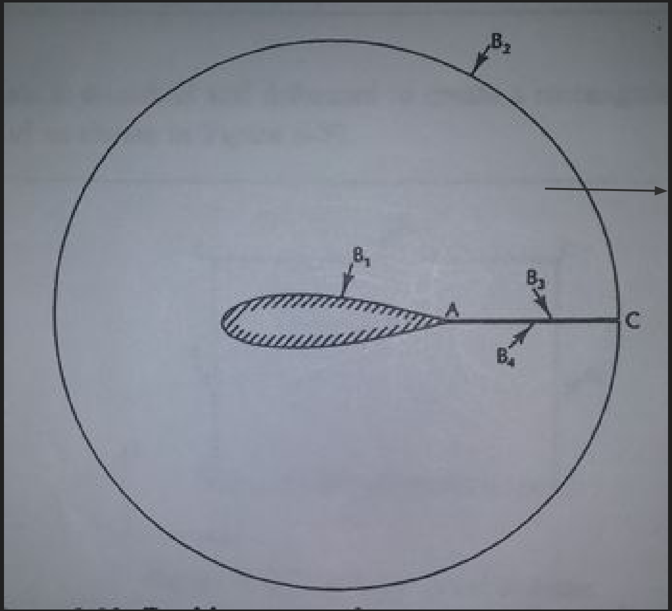




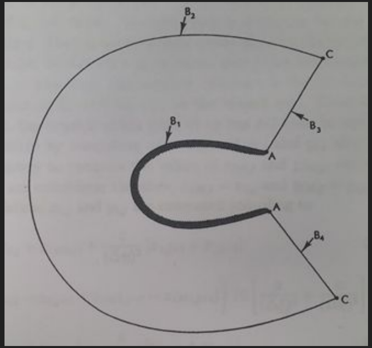
Boundary Value Problem - Solved Numerically

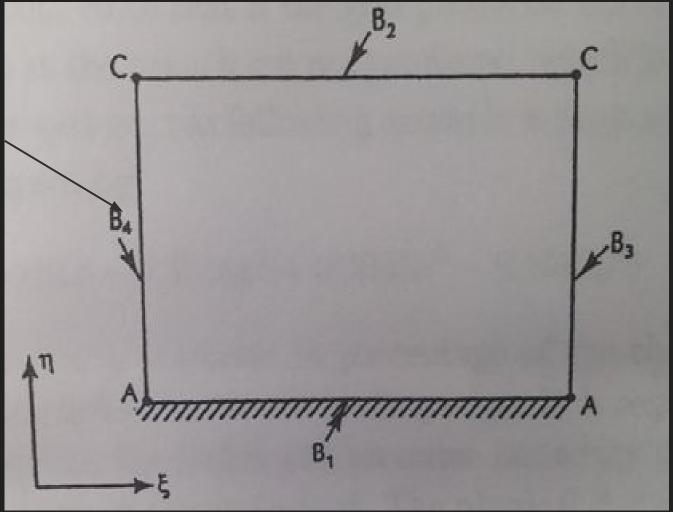
Joukowski Transformation (Complex mapping)

w =J(z)=z+k/z   ; z= x+iy



Physical Domain − xy plane





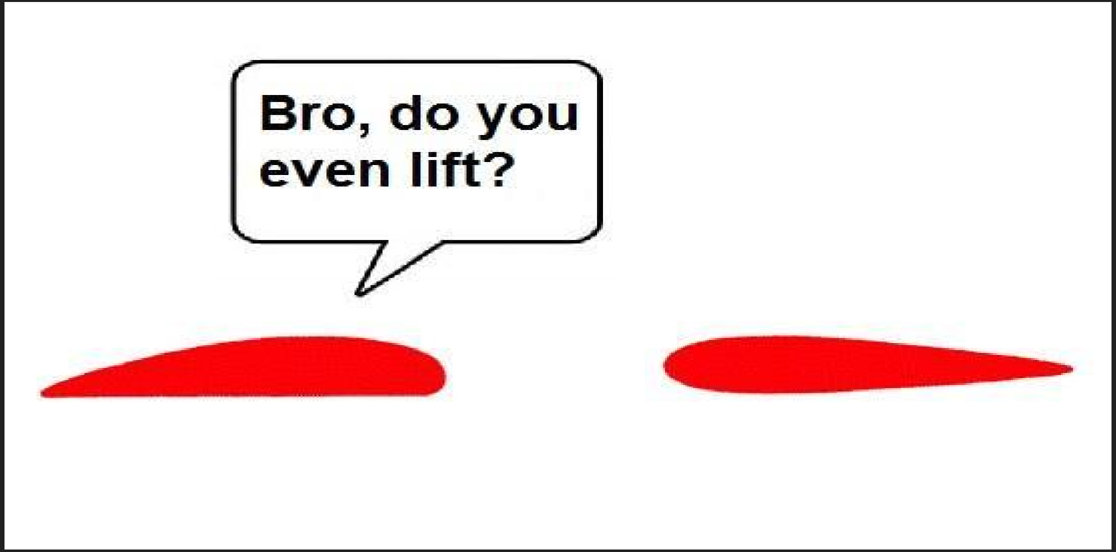
Computational Domain − ξη plane

Conclusion

Using splines to generate airfoils, we can experiment with new and different airfoil shapes

With computer software, we can test these new airfoils and determine if the new shapes do result in less drag and/or greater lift

WHAT DID THE CAMBERED AIRFOIL SAY TO THE SYMMETRICAL AIRFOIL?



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