Cats and Dogs: What's the Difference?

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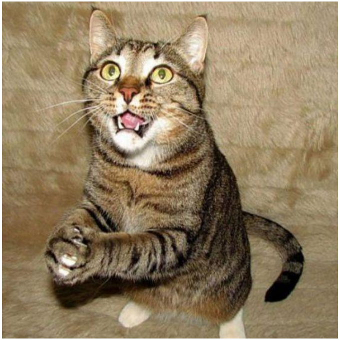
So the world may know

Outline

How to describe an object

What are the qualities that make up a cat or a dog?



(a) Typical Cat

(b) Typical Dog

Training Data and Classifying

Training

- Given a known set of data split into two classes
- Pick a method to test the data against itself
- Analyze how well it performs

Classify Unknowns

- Use training data against unknown data
- Have computer output predictions
- We then give results against actual values

Principal Angles

Given two subspaces compare the angles between them.

- Intuitively more similar subspaces should have a smaller angle between them.
- Method works by comparing each image to a gallery and seeing how close the angles get.
- The result with the smallest angle becomes the label of the unknown image.

Singular Value Decomposition

Refresher on what Singular Value Decomposition (SVD) is.

1. SVD is a means to turn matrix A into USV^T
2. For A (m by n) U is an orthogonal m by m matrix
3. V is an orthogonal n by n matrix
4. S is a m by n matrix of all zeros except for the main diagonal being the singular values of A^TA
5. Singular values are the square roots of the eigenvalues

How Principal Angles Works

1. Find orthonormal bases for input matrices X and Y labeled Q_X and Q_Y.
2. Find svd of cosine: perform svd on $Q_X^T Q_Y$, singular values list as $(\sigma_1, \sigma_2, \cdots, \sigma_n)$
3. $Y = \begin{cases} Q_Y - Q_X(Q_X^T Q_Y) & \text{if } rank(Q_X) < rank(Q_Y) \\ Q_X - Q_Y(Q_Y^T Q_X) & else \end{cases}$
4. Find svd of sine: perform svd on Y, singular values list as $(\mu_1, \mu_2, \cdots, \mu_m)$
5. k^{th} angle is given by $\theta_k = \begin{cases} \arccos(\sigma_k) & \text{if } \sigma_k^2 < 0.5 \\ \arcsin(\mu_k) & \text{if } \mu_k^2 \leq 0.5 \end{cases}$ for k from 1 to $min(rank(Q_X, Q_Y))$

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How to classify using Principal Angles

1. Test sample image against the cat gallery and the dog gallery using Principal Angles.
2. Label test image as a cat or dog depending on which angle was smaller.
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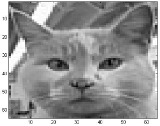
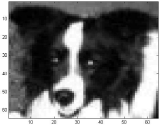
Results of using Principal Angles

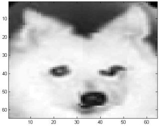
Principal Angles classified correctly $\frac{33}{38}$ of our test images. Below are the five it missed.



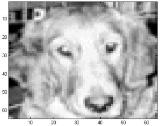
(c) Image 4



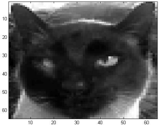
(d) Image 17



(e) Image 19



(f) Image 20



(g) Image 34

Principal Component Analysis

1. Calculate Ensemble Average and mean subtract the data.
2. Perform SVD of mean subtracted data.
3. Calculate D(dimension) retaining 99% energy.
4. Reduce the dimensions of KL basis and the coefficients of the gallery using D.

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PCA/Principal Angles

- Using reduced dimension data, run through Principal Angles again.
- Eigenanimals show the main characteristics of the animals.
- Results are slightly improved.

Eigenanimals

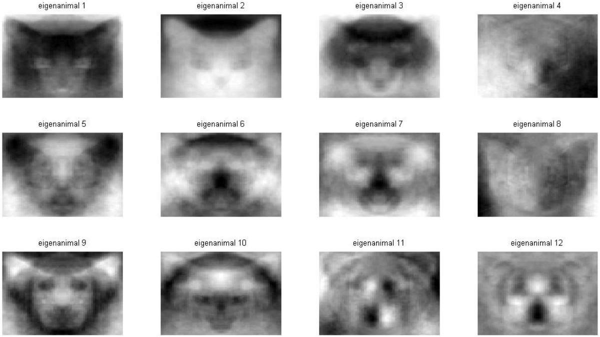


Figure: The first 12 eigenanimals displaying the top 12 characteristics found in the images.

Results of using modified Principal Angles

Modified Principal Angles classified correctly $\frac{35}{38}$ of our test images. Below are the three it missed.

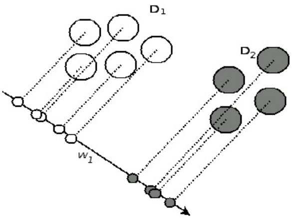
(a) Image 4

(b) Image 17

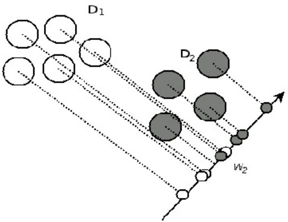
(c) Image 34

Fisher Discriminant Analysis

- Fisher Discriminant Analysis is a classification method that finds an optimal projection to one dimension and projects all the data onto that line.
- The goal is to separate the training data completely on the projection so that we can pick a nice threshold value.
- This threshold value separates the cats and the dogs.



(d) Good Projection



(e) Bad Projection

How FDA works

m_1 and m_2 are the class-wise means.

1. Create the between-class scatter matrix, $S_b = (m_2 - m_1)(m_2 - m_1)^T$.
2. Create the within-class scatter matrix, $S_w = \sum_{i=1,2} \sum_{x \in D_i} (x - m_i)(x - m_i)^T$.
3. Find the optimal projection ω by simultaneously maximizing the between-class scatter and minimizing the within-class scatter.

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How to classify using FDA

Project unknown data to the real line, classify with regard to threshold value.

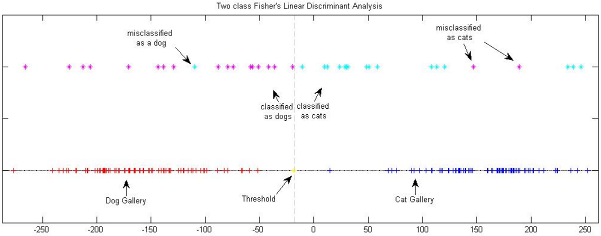


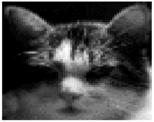
Figure: Two class FDA. Dogs from the probe are in magenta, while cats from the probe are in cyan.

Results of using FDA

FDA classified correctly $\frac{35}{38}$ of our test images. Below are the three it missed.

(a) Image 17

(b) Image 18



(c) Image 23

What is a wavelet?

1. Wavelets are what they sounds like - small waves.
2. Mother wavelets have a fixed shape and period.
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Haar Mother Wavelet

The wavelet used for this test was the Haar wavelet, or just a square wave.

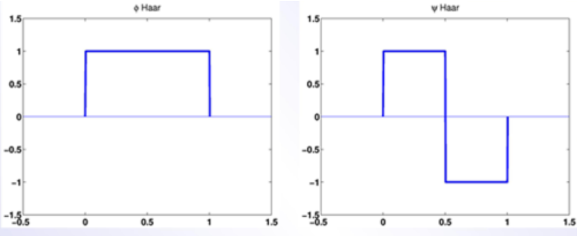


Figure: Mother Haar

What Wavelet Transforms do to an image

- Applying a wavelet transform to an image decomposes it into four parts.
- These parts are a quarter of the original size.
- The four parts are the approximate, horizontal details, vertical details, and diagonal details.

Finding Edges With Wavelets

The horizontal and vertical details correspond to a low and high pass filter combined.

(a) A sample cat

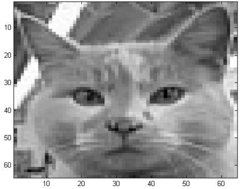
(b) Details

Wavelet Edge Method

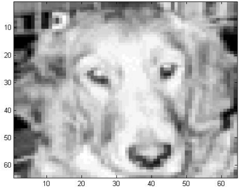
1. Run a wavelet transform on each image.
2. Add the horizontal and vertical details.
3. Run Principal Angles on this new image.

Results of using Wavelet Edge Method

Principal Angles classified correctly $\frac{36}{38}$ of our test images. Below are the two it missed.



(c) Image 4



(d) Image 19

Conclusion

1. We dressed up, so we deserve an A.
2. We had a method get 95% accuracy, so we deserve an A.
3. We are your favorite students, so we should all get A's.

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References

Jen-Mei Chang.
MATRIX METHODS FOR GEOMETRIC DATA ANALYSIS AND PATTERN RECOGNITION, 2009.

Figure: None of the cats were as good looking as this one.