

Math 695 Project: Motion Analysis

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Background of Motion Analysis

Motion Analysis is used to analyze qualitative and quantitative behaviors of objects in motion.

Some areas that uses Motion Analysis are:

- 1 Human Motion Analysis
- 2 Manufacturing
- 3 Military Uses

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- ① **Human Motion Analysis**
 - Automatic security monitoring
 - Gait or posture optimization
- ② Manufacturing
 - Analyze projectiles to test products
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 - Long range weapons like missiles

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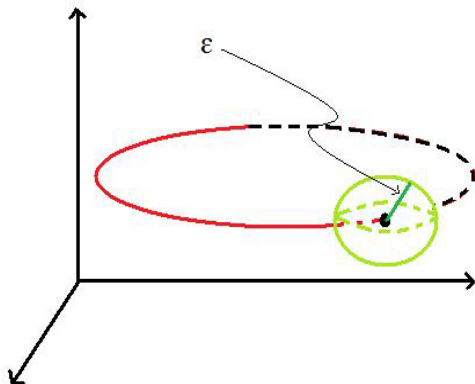
Introduction

Based on "Motion Analysis Using The Novelty Filter" by
E. Ardizzone, A. Chella, S.Gaglio, and F.Sorbello

- Use Novelty Filter and apply to a training set.
- Apply a new frame snapshot to the filter.

Introduction

- For a given threshold ε determine if the position is on the trajectory.
- If it is not on the trajectory then update the new training set.



Problems

Draw Backs: Lose some wanted data in 2D.

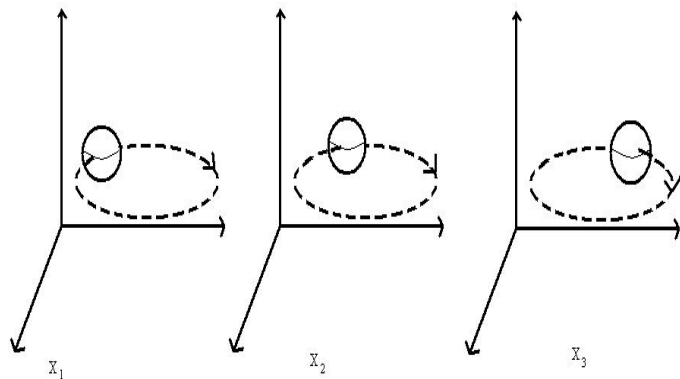
i.e.

- 1 <http://anthonydevito.net/orbit1.swf>
- 2 <http://anthonydevito.net/orbit2.swf>

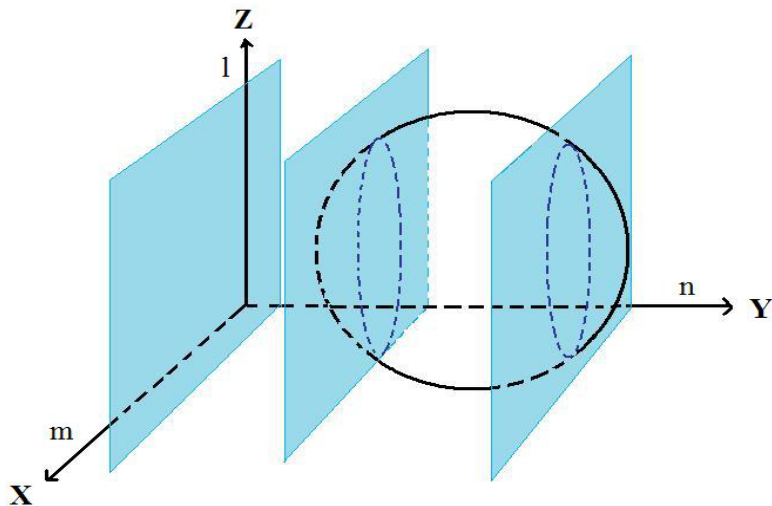
Proposed Modification: Do a 3D motion analysis with the same idea.

Setting Up Training Set

The training set will contain a set of 3D images.

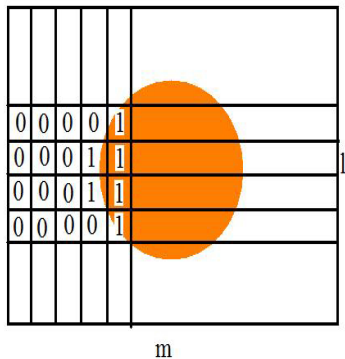


Slice it ALL Up



Each slice along the y will be a single 2D image.

A Single Slice



$$y_i \in \mathbb{R}^m, i = 1, 2, \dots, n$$

Concatenating the Concatenated

$$X_j = \begin{pmatrix} y_1 \\ y_2 \\ \vdots \\ y_n \end{pmatrix}$$

where $X_j \in \mathfrak{R}^{l \times m \times n}$, $j = 1, 2, \dots, t$

$$S = \{X_1 | X_2 | X_3 | \dots | X_t\} \text{ so } S \in \mathfrak{R}^{l \times m \times n \times t}$$

Kohonen's Novelty Filter

Based on **Geometric Data Analysis** by Michael Kirby

Let V be the subspace spanned by the training set.

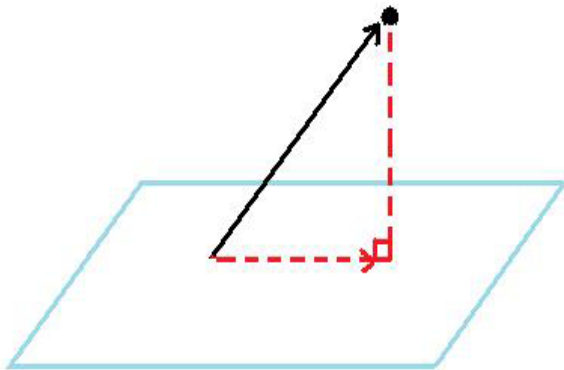
$$\begin{aligned} V &= \text{Span}\{X_1, X_2, \dots, X_t\} \\ &= \text{Span}\{m_1, m_2, \dots, m_t\} \end{aligned}$$

where $\{m_j\}_{j=1}^t$ is the orthonormal basis.

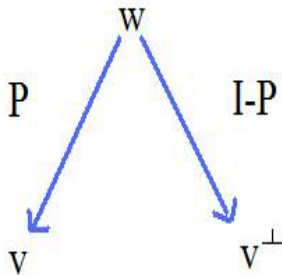
Seeking a Projection

- Let M be the matrix consisting of orthonormal vectors of S .
- We can find P , the **projection matrix**, by multiplying M and M^T .

$$P = M * M^T$$

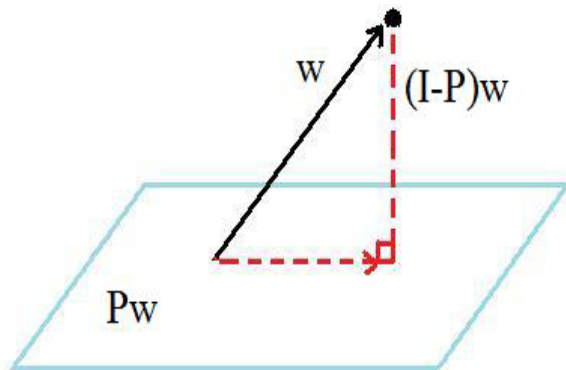


The Novel and the Not-so-Novel: I



- $Pw = n \in \mathbb{R}^T$
- $(I - P)w = n^\perp \in \mathbb{R}^{lmn-t}$

The Novel and the Not-so-Novel: II



$$w = n + n^\perp$$

Implementing

We want to find the **residual(novelty)** of each incoming snapshot $w \in \mathbb{R}^{lmn}$ by applying the matrix

$$E = I - P$$

$$\Rightarrow Ew = n$$

where $n \in \mathbb{R}^{lmn}$

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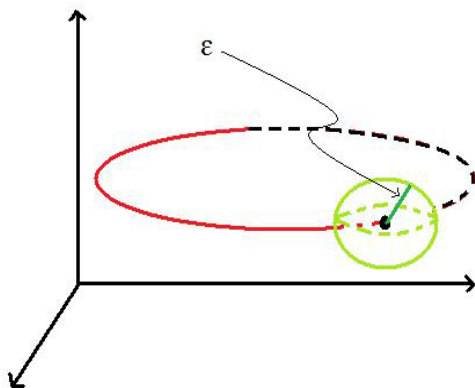
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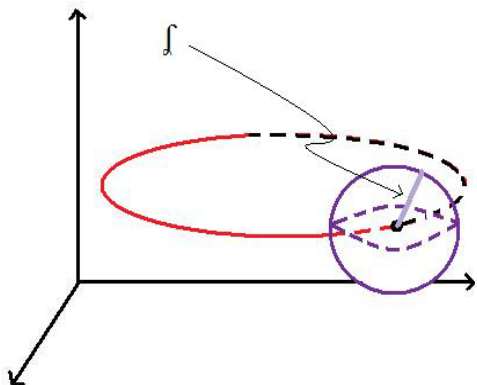
On or Off

- Determine where the new snapshot is already on the trajectory.
- If not, is it less than the set threshold?



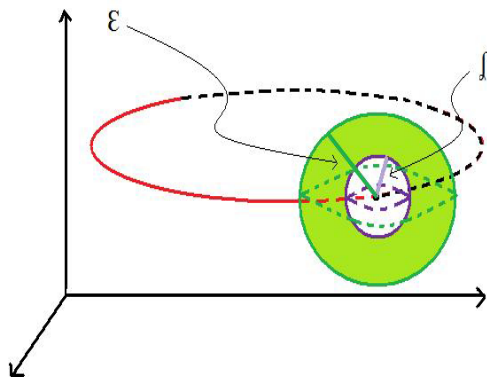
Case 1

If $\|n\| \leq \delta$, where δ is small almost zero, then w is on the trajectory.



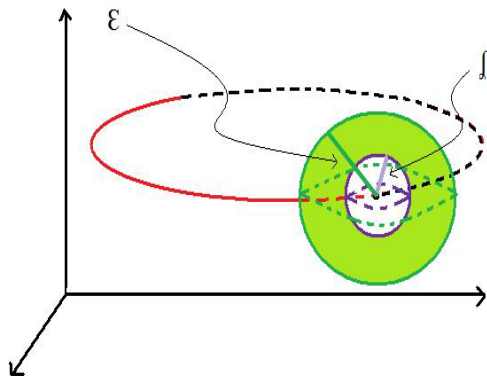
Case 2

If $\delta \leq \|n\| \leq \varepsilon$, where ε is the threshold, then w is not on the current trajectory but w can be taken as a new trajectory.

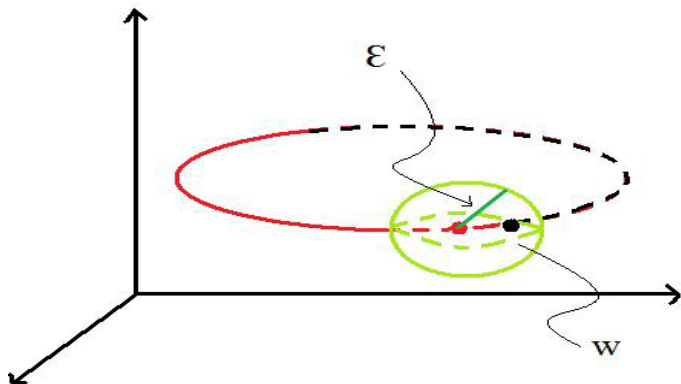


Case 2

For a more accurate trajectory ε must be small.



Case 2

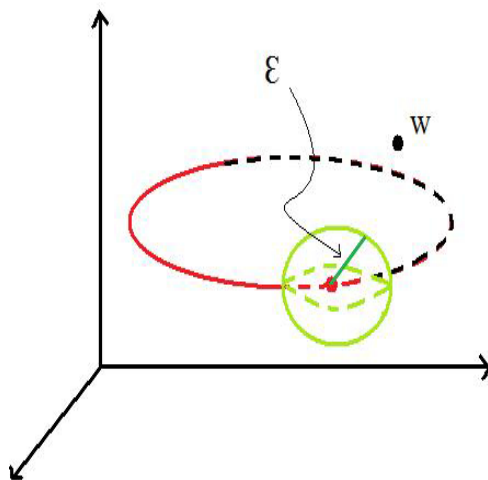


Update the training with this new w .

$$S_{old} = \{X_1 | X_2 | X_3 | \dots | X_t\}$$
$$S_{new} = \{X_1 | X_2 | X_3 | \dots | X_t | w\}$$

Case 3

If $\|n\| \geq \varepsilon$ then disregard w , otherwise the trajectory will be inaccurate.



Pseudocode

1. Initialize all given data: $\delta, \epsilon, X_1, X_2, \dots, X_t, w$.
2. Create $S = \{X_1 \mid X_2 \mid X_3 \mid \dots \mid X_t\}$ via horizontal concatenation.
3. Orthonormalize S via Gram-Schmidt or QR ($M = \text{orth}(S)$).
4. Create $P = M * M^T$.

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5. Create $E = I - P$.
6. Find Novelty: $Ew = n$.
7. If $\|n\| \leq \delta$, w is on trajectory. Done.
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Pseudocode

9. If $\delta \leq \|n\| \leq \varepsilon$

10. Vertical concatenate w with old training set.

$$S = \{X_1 \mid X_2 \mid X_3 \mid \dots \mid X_t \mid w\}$$

11. Repeat 3-10 with every new snapshot.

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Pros and Cons of 2D Case

Pros:

- Cheap and easy to compute
- Data is easy to obtain

Cons:

- Lose some wanted information

Pros and Cons of 3D Case

Pros:

- Maintains all necessary information

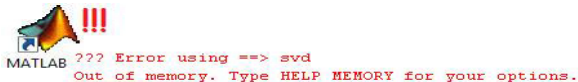
Cons:

- Expensive to compute
- Data is hard to acquire

Fear the "Ginormous" Matrix

The matrix $P = M * M^T \in \mathbb{R}^{lmn \times lmn}$ is extremely large.

For example: take a snapshot to be 100x100x100 where each slice is a 100x100 picture. then $P \in \mathbb{R}^{1,000,000 \times 1,000,000}$



Do you have it in X-Small?

1. We can reduce the size of P by doing a Pyramidal Decomposition.
 - Decompose every news incoming snapshot to the same level.
 - Proceed to analyze motion via Novelty Filter on the approximation portion.

Do you have that in X-Small?

2. Modify the size of P by using Singular Value Decomposition (SVD) on S.

- Use SVD on $(S^T * S) \in \mathbb{R}^{txt}$ and orthonormalize the bases.
- Do the analysis using this new P and project back up to original dimension.

Predictions

Can you predict the entire trajectory with the training set? No.

However, we can tell if a guessed position will be on the trajectory or not by calculating

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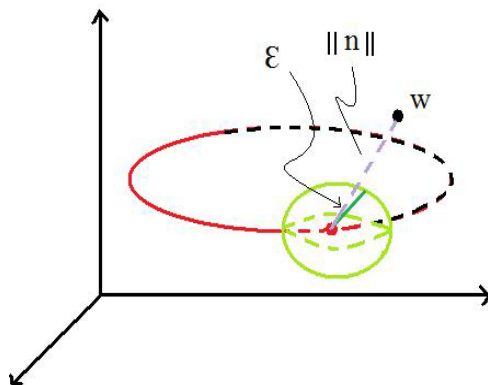
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$$\frac{\varepsilon}{\|n\|}$$

Predictions



If $\frac{\epsilon}{\|n\|}$ is small then there is a low probability that the guessed position will be on the trajectory and vice versa.

Conclusion

- Given a training set of positions of an object
- Use novelty filter to find the difference between the training set and the new snapshot
- Analyze whether the new snapshot can be added onto the trajectory
- Update the new trajectory

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