# Cats vs. Dogs 

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## Outline

(1) Introduction
(2) Methods

- Wavelet Analysis
- Principal Angles
- Kohonen's Novelty Filter
- Kernel Linear Discriminant Analysis
(3) Results

4. Conclusion

## Introduction

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## The data



Figure: Images of cats and dogs

## Eigencat and Eigendog

Since eigenvectors are an important aspect of all our methods, we feel it is important to explore the eigencat and eigendog.


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## Wavelet Analysis

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## The equations

Define the scaled and translated basis functions as:

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\begin{aligned}
\phi_{m, n}^{j}(x, y) & =2^{-j / 2} \phi\left(2^{-j} x-m, 2^{-j} y-n\right) \\
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The discrete wavelet transform of an image $f(x, y)$ of size $M \times N$ is then:

$$
\begin{aligned}
c_{m, n}^{j} & =\frac{1}{\sqrt{M N}} \sum_{x=0}^{M-1} \sum_{y=0}^{N-1} f(x, y) \phi_{m, n}^{j}(x, y) \\
\left(d_{m, n}^{j}\right)^{i} & =\frac{1}{\sqrt{M N}} \sum_{x=0}^{M-1} \sum_{y=0}^{N-1} f(x, y)\left(\psi_{m, n}^{j}(x, y)\right)^{i}
\end{aligned}
$$

where $i=\{H, V, D\}$.

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## Decomposition of a cat and dog



Figure : 2 level decomposition of a cat (left) and dog (right).

## Principal Angles - Introduction

- A projection, in $\mathbb{R}^{2}$, is a transformation of a vector into a different vectors direction.

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## Principal Angles

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- Principal angles is applying the $\cos (\theta)$ formula for all combinations of vectors among the two vector spaces.
- The smaller the $\theta$ the more "similar".


## Putting it all together

$M$ is the training sets, and $N$ is the test set.

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## Kohonen's Novelty Filter

- Kohonen's way to compare the two images and see the difference.

The process is to pick the best characteristic from each of the test sample by using Singular Value Decomposition. The imace that we want to test will he nroiected to the Singular Value Decomposition and subtract from the original, whichever give the smallest two norms (implies smaller difference from the Sinaular Value Decomoosition) will be categorize to be the same class with the image.

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\begin{aligned}
& X=S C \times S C^{\prime} \times T T(:, i)-T T(:, i) \\
& Y=S D \times S D^{\prime} \times T T(:, i)-T T(:, i) \quad i=1,2, \ldots K,
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If $\|X\|_{2} \leq\|Y\|_{2}$ the test consider it to be a cat and it is a dog otherwise.
The reason why the filter pick SC or $S D$ is because $S C \times S C^{\prime}=I$, where I is the identity matrix. As a result after we apply the filter, we should get the same (or close to the original vector) if they are from the same type.

## KLDA - Introduction

- KLDA generalizes LDA since in the transformed space, the principal components are nonlinearly related to the input variables.

Kernel Linear Discriminant Analysis (KLDA) maps the input space into a high dimensional, nonlinear feature space This transformation is carried out by a kernel function
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- Kernel Linear Discriminant Analysis (KLDA) maps the input space into a high dimensional, nonlinear feature space. This transformation is carried out by a kernel function $\phi: X \rightarrow F$.
- Common kernel function is the RBF (Gaussian): $\phi(\mathbf{x}, \mathbf{y})=\exp \left(-\frac{\|\mathbf{x}-\mathbf{y}\|^{2}}{2 \sigma^{2}}\right)$ where the assumption is that the classes have a multivariate Gaussian distribution.


## KLDA - Intro. cont



Figure : Feature space transformation.

## Implementation

- After mapping data to feature space, then same procedure as LDA to find optimal direction that separates classes:
- Rayleigh quotient: $J(\mathbf{w})=\frac{\mathbf{w}^{\top} S_{b}^{d} \mathbf{w}}{\mathbf{w}^{\top} S_{w}^{d} \mathbf{w}}$.
- Solve generalized eigenvalue problem: $S_{B}^{\phi} \mathbf{w}=J(\mathbf{w}) S_{W}^{\phi} \mathbf{w}$.


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- Solve generalized eigenvalue problem: $S_{B}^{\phi} \mathbf{w}=J(\mathbf{w}) S_{W}^{\phi} \mathbf{w}$.
- Performed leave-one-out cross-validation (LOOCV) on training set to find best parameter $\sigma$ for kernel function and energy for dimensionality reduction using principal components analysis (PCA) ([1],[3],[4]).


## Method of testing

- For each method we run it through cross-validation. training images in order to find the "optimal" number of training imades in order to produce the best results. Logically, it would seem that using all images as a training set would be the best, but if we could produce the same results with half as manv, then the time it takes will be reduced.


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- After all the data is collected, we average the results.


## Final results

Our testing data consists of 38 images, 19 cats and 19 dogs. As stated before, we ran our algorithms with a varying number $(40,50,60,70)$ of training images.

For each case, we ran 11 iterations cycling through all the images to make sure all images were included at least once in our training set.

For the methods that require cumulative energy, we used $99 \%$.

## Final Results

| Method | 40 | 50 | 60 | 70 |
| :---: | :---: | :---: | :---: | :---: |
| Principal Angles | .8684 | .8421 | .8684 | .8684 |
| Novelty Filter | .9211 | .9211 | .9211 | .8947 |
| LDA | .8158 | $\mathbf{. 8 4 2 1}$ | $\mathbf{. 8 4 2 1}$ | $\mathbf{. 8 4 2 1}$ |

Table : Best results from using original data.

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Table : Best results from using first wavelet approximation.

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| LDA | .7632 | .8158 | .7632 | .7632 |

Table : Best results from using first wavelet horizontal detail.

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| Principal Angles | .9211 | .9211 | .8947 | .8947 |
| Novelty Filter | .9474 | .9211 | .9211 | .8947 |
| LDA | .7105 | .6579 | .6579 | .7368 |

Table : Best results from using first wavelet vertical detail.

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| LDA | .7105 | .6842 | .6579 | .6053 |

Table : Best results from using second wavelet approximation.

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| LDA | .7895 | .8684 | .8158 | .7895 |

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## Final Results

| Data Set | $\boldsymbol{\sigma}$ | e | Accuracy |
| :---: | :---: | :---: | :---: |
| Training | 6.05 | 0.75 | $0.89375(143 / 160)$ |
| Validation | 5.45 | 0.75 | $0.92105(35 / 38)$ |

Table : KLDA classification performance.

## Final Results



Figure : Separation using KLDA

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- KLDA demonstrated best separation of classes over PCA and KPCA, but did not project well.
- MidRange (nonparametric) threshold classification boundary performed better than (parametric) Mahalanobis distance. This is an indicator that the data may not be Normal.


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Comparing different kernel function(s).

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## Thank you



## References

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## Any questions?



