Mathematical Modeling
Math - 579

Daniel Macias, Mutaz Alsayegh,
Ramy Heng, Rattana Hor

Problem

The goal of our project is to convert a 2D image of a placenta into a 3D object using Matlab.

Purpose

- Be able to predict the configuration of the vessel network based on the 3-D geometry.
  - The density of the vessel network is directly correlated to the health and development of the fetus.
    - Vessel network provides oxygen and nutrition to the developing fetus

Plan of Action

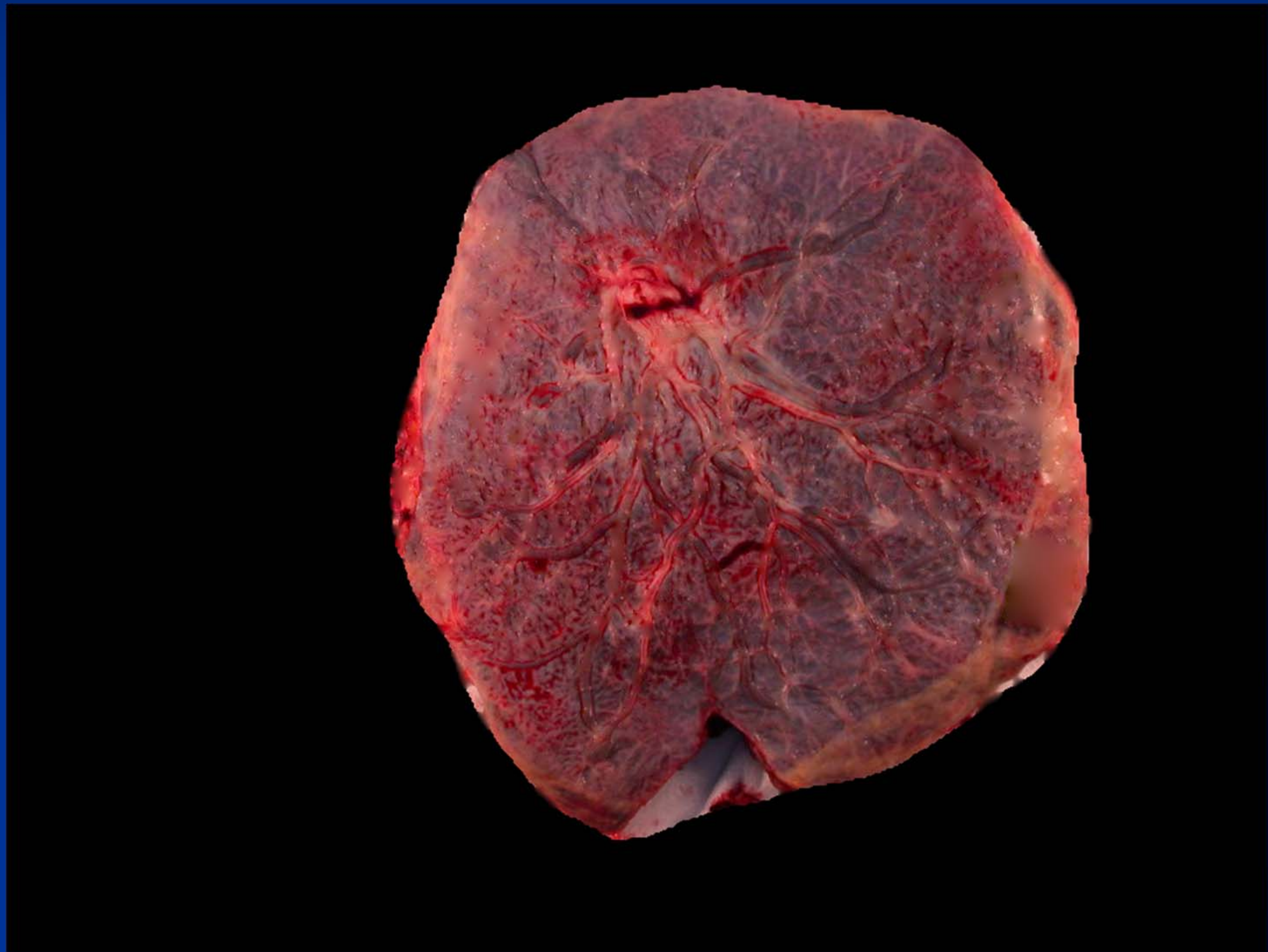
- Obtain a cropped image of our placenta
- Filter the cropped image to get a better view of the vessel network and to eliminate any unwanted data.
- Create a point cloud from that filtered image.
- Use the cloud as a basis for our 3D object.
- Interpolate our points
- Fit a polynomial over these points
- Create a Mesh over our polynomials to create texture.
- Use the data we have from our slice to give our cloud some volume.

Cropping

- The first step in this process is to crop the 2D image to eliminate any unwanted objects that might be of less use for our purposes.
- Our original plans were to implement this ourselves but Dr. Chang was kind enough to lend us her code and images.

Original Slide: Placenta #1786

Cropped Slide: Placenta #1786



Filtering

We used an Averaging Filter method that is used for blurring and noise reduction.

Filter (mask, kernel, template or window):

- Averaging filter:

One type of spatial domain filter (image plane itself or aggregate of pixels composing an image) that is used for blurring and for noise reduction.

- The output (response) of averaging filter:

The average of the gray levels (intensities) in the neighborhood defined by the filter mask.

Ex: 3-by-3 averaging filter mask

$\frac{1}{9} \times$

1	1	1
1	1	1
1	1	1

$R = \frac{1}{9}\sum_{i=1}^{9} w_i z_i$, where R is the output or response

w's are mask coefficients

z's are the values of image gray levels corresponding to those

$$R = \frac{1}{9}\sum_{i=1}^{9} z_i$$

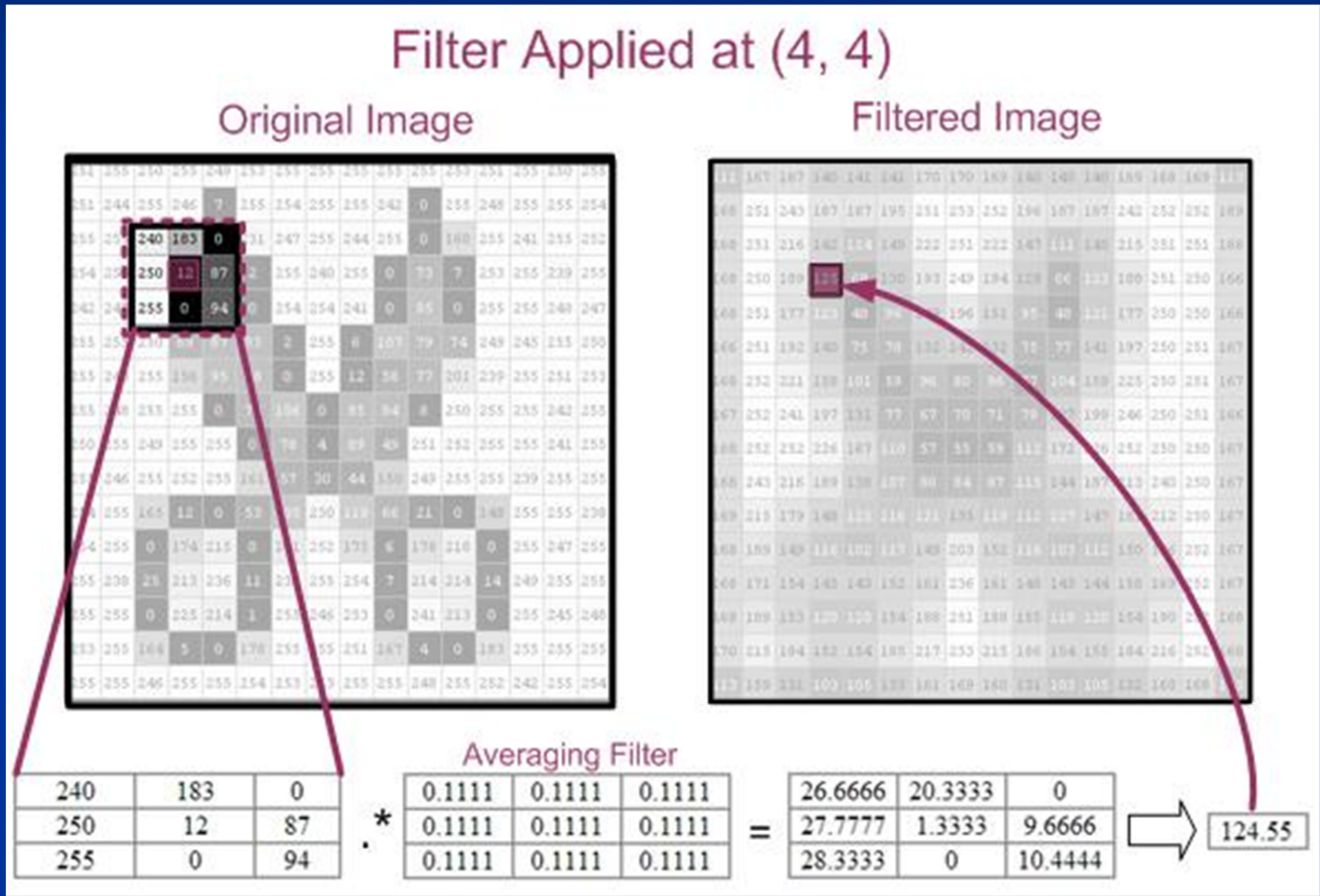
General implementation for filtering an image f of size $M \times N$ with a weighted averaging filter of size $m \times n$ (m and n odd)

$$g(x,y) = \frac{\sum_{s=-a}^{a} \sum_{t=-b}^{b} w(s,t) f(x+s, y+t)}{\sum_{s=-a}^{a} \sum_{t=-b}^{b} w(s,t)}$$

- $a = (m-1)/2$ *and* $b = (n-1)/2$
- $x = 0,1,2,\ldots,M-1$ *and* $y = 0,1,2,\ldots,N-1$

How the filter works

Averaging filter applied to position (4,4)



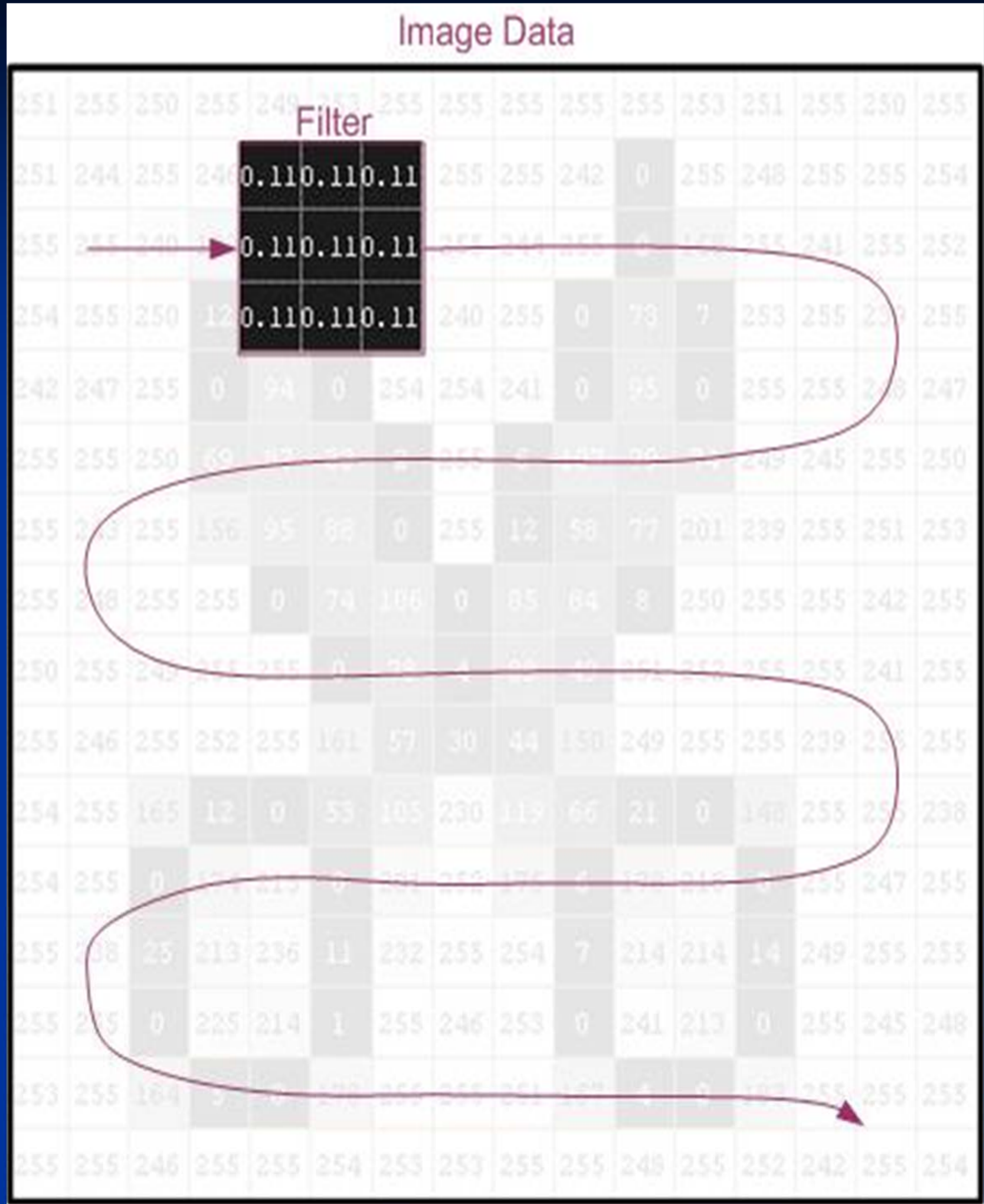


Image Data
Filter
0.11 0.11 0.11
0.11 0.11 0.11
0.11 0.11 0.11

Boundary Problems

Filter Applied at 1,1

251	255	250	255	249	253	255	255	255	255	255	253	251	255	250	255
251	244	255	246	7	255	254	255	255	242	0	255	248	255	255	254
255	255	240	183	0	231	247	255	244	255	0	168	255	241	255	252
254	255	250	12	87	2	255	240	255	0	73	7	253	255	239	255
242	247	255	0	94	0	254	254	241	0	95	0	255	255	248	247
255	255	250	69	87	83	2	255	6	107	79	74	249	245	255	250
255	243	255	156	95	88	0	255	12	58	77	201	239	255	251	253
255	248	255	255	0	74	106	0	85	84	8	250	255	255	242	255
250	255	249	255	255	0	78	4	89	49	251	252	255	255	241	255
255	246	255	252	255	161	57	30	44	150	249	255	255	239	255	255
254	255	165	12	0	53	105	230	119	66	21	0	148	255	255	238
254	255	0	174	215	0	201	252	175	6	178	218	0	255	247	255
255	238	25	213	236	11	232	255	254	7	214	214	14	249	255	255
255	255	0	225	214	1	255	246	253	0	241	213	0	255	245	248
253	255	164	5	0	178	255	255	251	167	4	0	183	255	255	255
255	255	246	255	255	254	253	253	255	255	248	255	252	242	255	254

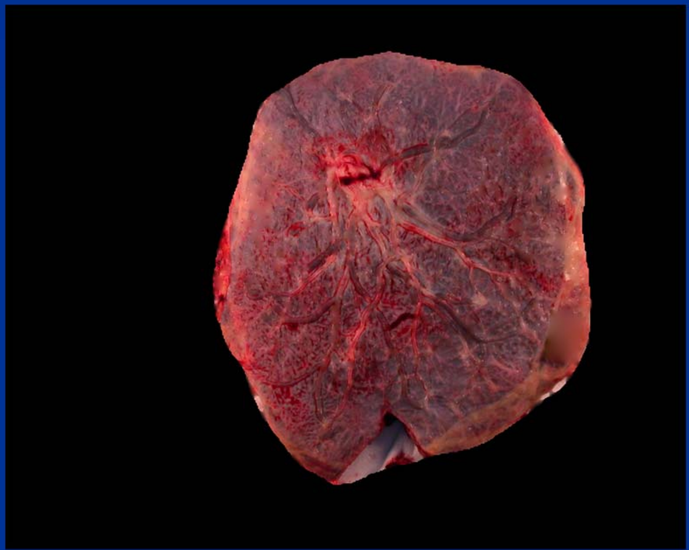
Solution

Zero Padding

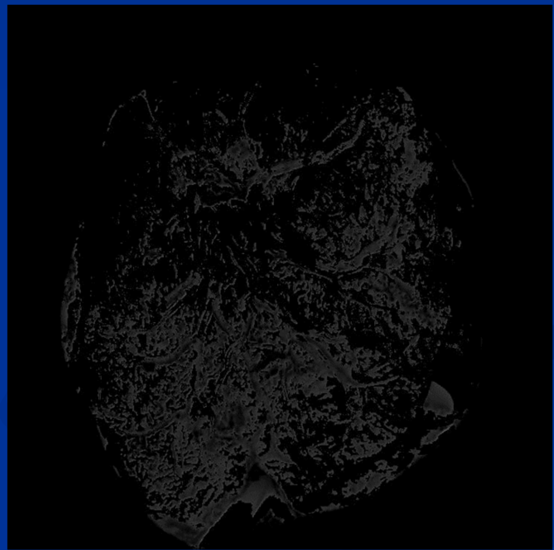
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	251	255	250	255	249	253	255	255	255	255	255	253	251	255	250	255	0
0	251	244	255	246	7	255	254	255	255	242	0	255	248	255	255	254	0
0	255	255	240	183	0	231	247	255	244	255	0	168	255	241	255	252	0
0	254	255	250	12	87	2	255	240	255	0	73	7	253	255	239	255	0
0	242	247	255	0	94	0	254	254	241	0	95	0	255	255	248	247	0
0	255	255	250	69	87	83	2	255	6	107	79	74	249	245	255	250	0
0	255	243	255	156	95	88	0	255	12	58	77	201	239	255	251	253	0
0	255	248	255	255	0	74	106	0	85	84	8	250	255	255	242	255	0
0	250	255	249	255	255	0	78	4	89	49	251	252	255	255	241	255	0
0	255	246	255	252	255	161	57	30	44	150	249	255	255	239	255	255	0
0	254	255	165	12	0	53	105	230	119	66	21	0	148	255	255	238	0
0	254	255	0	174	215	0	201	252	175	6	178	218	0	255	247	255	0
0	255	238	25	213	236	11	232	255	254	7	214	214	14	249	255	255	0
0	255	255	0	225	214	1	255	246	253	0	241	213	0	255	245	248	0
0	253	255	164	5	0	178	255	255	251	167	4	0	183	255	255	255	0
0	255	255	246	255	255	254	253	253	255	255	248	255	252	242	255	254	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0

Results:

Original image



Filtered image



Data Provided By Dr. Chang

- Along with pictures (cropped, sliced and de-glared), Dr. Chang provided us with mountains of data.
- Each placenta was cut into anywhere from 7 to 17 slices.

Placenta Data

Each sliced specimen came with measurements

A	B	C	D	E	F	G	H	I	J	K	L	M
ID	nRows_1	Pitch_1	MinMaxPitch_1	Taper_1	RngY_1	Area_1	Perim_1	AvgThick_1	MinThick_1	MaxThick_1	MinDivMax_1	SDThick_1
1768	12	0.781055641	0.33838475	0.36525118	8.509657383	2.55519779	17.83903401	0.287325578	0.146070805	0.524644705	0.278418525	0.153434687
1769	13	0.989217151	0.845000765	0.627134309	11.48281956	4.846047543	23.50037341	0.394336484	0.237132004	0.849280702	0.27921511	0.242010373
1770	18	1.131232619	0.676058297	0.075512399	19.63375092	26.38762502	43.77517138	1.308299705	0.460291352	2.05912075	0.223537814	0.643030682
1771	14	0.870450934	0.90769192	0.299615714	11.52492523	7.080063402	23.8009314	0.583869295	0.370776697	1.219725892	0.303983624	0.319031792
1772	7	0.904699326	0.812500844	0.295265675	5.955482483	8.313896424	13.51332432	1.362397877	1.353893285	1.945216019	0.696011791	0.679608973
1773	15	0.786382675	0.815725391	0.184136424	11.2689395	7.494705242	23.04705632	0.63993667	0.689108987	0.977426354	0.705023948	0.288550129
1774	16	0.848657199	0.832924194	0.522519325	12.88547039	8.651168019	26.81853389	0.64080279	0.38251201	1.139795723	0.335596987	0.31838531
1776	17	1.175917753	0.449664658	0.142056354	18.33163357	12.81948361	37.86225877	0.678139373	0.307400878	1.405090207	0.218776614	0.343930991
1777	8	0.804681619	0.836984109	0.208199227	5.379439354	3.209905526	11.52562291	0.538086261	0.631124148	0.770820139	0.818769666	0.26766758
1778	16	0.880796603	0.78117513	0.156167778	13.49370956	14.13516114	28.31714704	1.016120675	0.777083733	1.496305802	0.519334839	0.406854464
1782	13	0.879934658	0.776190472	0.324550186	10.97204685	9.0135057	24.96275852	0.811174452	0.563516244	1.582106658	0.356180944	0.400101988
1783	12	0.96584053	0.837091933	0.123500423	10.97217464	7.316033617	23.15046453	0.641750484	0.582852681	1.112967775	0.523692324	0.270787929
1785	11	1.79889022	0.296985612	0.638746968	16.87928391	18.90012999	35.90341095	1.132773664	0.265415469	2.217122692	0.119711674	0.773470036
1786	10	0.923927903	0.876235631	0.284108065	8.068050385	4.199205379	16.45406283	0.474824656	0.572604286	0.834395574	0.686250387	0.267344267
1787	13	0.851054972	0.857142253	0.275217359	10.40429783	10.08090339	21.5571241	0.925124856	0.960219225	1.486062564	0.646149932	0.460000064
1789	11	0.799562666	0.790865897	0.468678643	8.477468491	11.47083419	18.98718112	1.33673692	1.104924768	2.106516394	0.524527021	0.640689692
1790	14	0.879976749	0.752313562	0.267396317	11.37724876	7.563735564	23.56053704	0.621817698	0.623923322	1.006613491	0.61982412	0.2808906
1791	11	0.907925606	0.781022797	0.289822591	9.713781357	5.445053303	20.2019131	0.557994602	0.459499334	0.90736175	0.506412502	0.23961437
1792	10	0.79420948	0.827668348	0.36228761	6.991206169	3.623426349	14.76349707	0.471642972	0.469673418	0.718519806	0.653668019	0.21970583
1793	16	0.793491534	0.795181187	0.222807018	12.22878218	20.72809147	26.13704232	1.633299384	1.699158593	2.644143498	0.642612095	0.671985386
1794	20	0.802985695	0.744779525	0.293247782	15.34880257	30.37423301	32.55485612	1.909424514	1.756586118	2.946366756	0.59618719	0.80611276
1796	11	0.971063932	0.895406932	0.274432103	9.844697952	11.10404712	20.68732773	1.059018231	1.294026613	1.556505467	0.831366571	0.481524018
1797												
1798	9	0.867426464	0.776675732	0.214011611	7.303202629	5.679938481	15.17463761	0.746648706	0.78487494	1.217245047	0.644796167	0.372031478
1799	20	0.880747159	0.852130091	0.256550194	17.18297338	28.84703252	38.18778183	1.654569302	1.268650558	2.590399454	0.489750936	0.700866715
1800	10	0.933793664	0.817517076	0.276872011	9.096633911	4.700477207	18.50410397	0.502704687	0.419684035	0.815654577	0.51453648	0.294607404
1802	24	0.811334848	0.734882896	0.174592227	18.8313849	43.44294755	40.34819287	2.234500849	1.868388846	2.884332342	0.647771693	0.80789509
1803	8	0.882977804	0.901515082	0.245517123	6.83747673	3.656203238	14.12972997	0.526969227	0.575926481	0.895766068	0.642942953	0.26855068
1804	9	0.810118811	0.895000082	0.22218879	7.215448856	8.820241564	17.66080716	1.214938081	1.282149846	1.73503733	0.738975366	0.5775014
1806	9	0.813814436	0.931936332	0.228185996	7.312740326	4.937072229	15.02180525	0.6747067	0.636692167	0.987794939	0.644559049	0.288851085
1808	8	1.012856801	0.89168681	0.227597089	6.857517242	5.19920984	14.18589235	0.676052701	0.827780332	1.002753476	0.825507317	0.357579978
1809	17	1.030189069	0.73120765	0.397789194	17.05988932	11.52409261	34.94640822	0.663768908	0.426023212	1.030301134	0.413493879	0.30881423
1810	13	0.860931743	0.904638734	0.204981756	10.04649925	11.87102175	21.5752921	1.086815198	0.962622881	1.851388286	0.519946512	0.58861861
1812	10	1.039386392	0.466045603	0.287100911	9.703005791	5.902369303	20.05524642	0.549480511	0.563763518	1.181104063	0.477319091	0.374472626
1813	11	0.884954558	0.721854163	0.16395467	9.580183983	7.431171967	21.9283782	0.764768136	0.697178877	1.225470525	0.568907096	0.349236845
1814	11	0.997078472	0.860350181	0.128527554	9.62736702	16.65433799	24.21322816	1.570200765	1.401622832	2.206873741	0.635116911	0.721858904
1815	21	0.975411189	0.740318203	0.236984957	19.63567829	31.92504388	41.33833321	1.576767129	1.028672876	2.259227348	0.455320655	0.546094784
1816	11	0.998323229	0.696232166	0.363793926	10.58065319	5.971874949	21.7246985	0.561716665	0.399522794	0.899975126	0.443926484	0.307971772

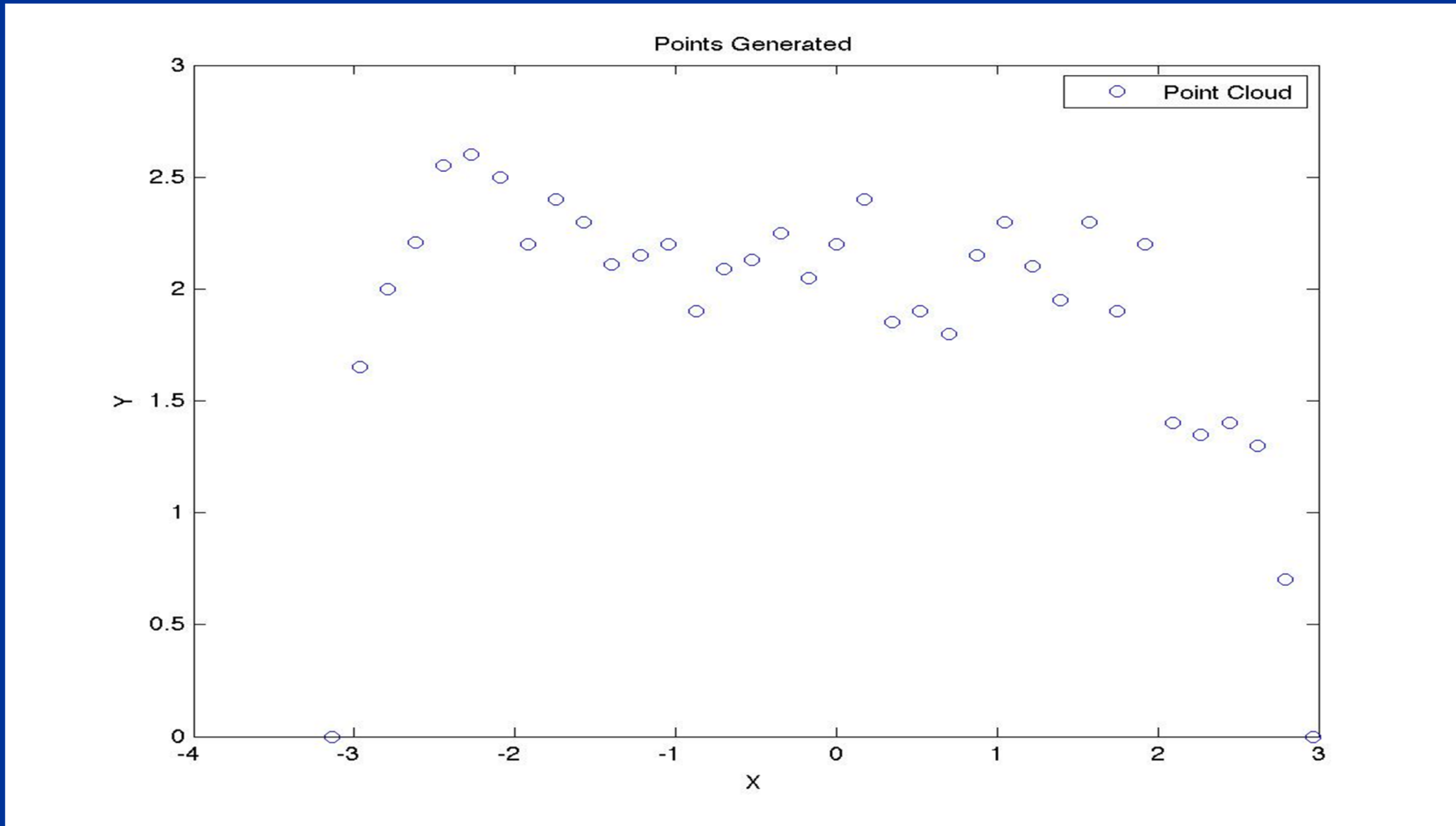
Data

- The data contained the following information:
  - Area, Perimeter, Average Thickness, Max and Minimum Thickness, Standard Deviation of the Measured Thickness, the Range of Y and so on….
- One data set we were hoping to extract was information about thickness and length of each slice.
- However the only information regarding the thickness that we were provided with was the average thickness of each slice.

Our New Approach

- Create a uniform mesh throughout the picture.
  - Manually measure the coordinates of the boundary of each slice.

- We divided each slice into an upper and lower surface.
  - For each section we stored its corresponding information in vector form.

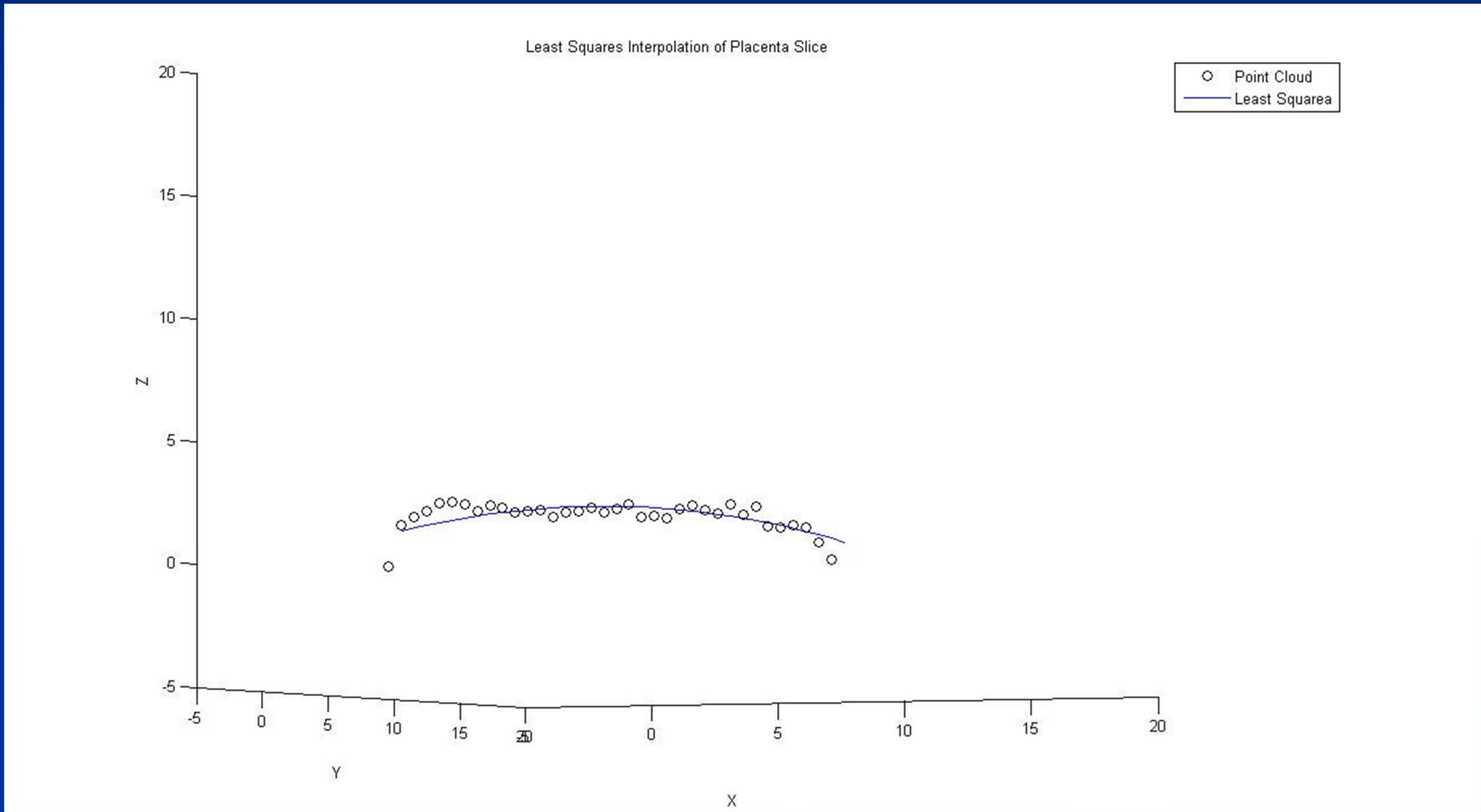


- We plan to interpolate each slice
    - Assemble the top surfaces together
    - Assemble the bottom surfaces together
- This renders a mesh of polynomials of the top and bottom surfaces of placenta.
    - Create large set of point cloud to be fitted with a mesh

Interpolating Our New Data

- We began experimenting with the interpolation of our point cloud by way of various methods..
- One method we tried was a Least Squares Polynomial Approximation

Point cloud of one slice with a least square fit:



Interpolation

- Suppose that a collection of 2m paired data points are given on the interval I = [-Π, Π]

$$\{(x_j, y_j)\}_{j=2}^{2m-1}$$

- For convenience we consider a portion on the interval, as follows:

$$x_j = -\pi + (j/m)\pi$$

For each j = 0,1,….,2m-1.

Interpolation

- For a fixed n<m, consider the orthogonal set

$$\tau_n = \{\phi_0, \ldots, \phi_{2n-1}\}$$

- Where:

$$\phi_0(x) = 1/2$$

$$\phi_k(x) = \cos(kx)$$

$$\phi_{n+k}(x) = \sin(kx)$$

Interpolation

- We want to find a trigonometric Polynomial S_n composed of functions from τ_n

that will minimize

$$E(S_n) = \sum_{j=0}^{2m-1} [y_j - S_n(x_j)]^2$$

- Hence we want to find constants

$$a_0, a_1, \ldots, a_n, b_1, \ldots, b_{n-1}$$

so that

$$E(S_n) = \sum_{j=0}^{2m-1} \left[y_j - [a_0/2 + a_n \cos(x_j) + \sum_{j=1}^{n-1} (a_k \cos(kx_j) + a_{n+k} \sin(kx_j))] \right]^2$$

is minimized.

Interpolation

- Here is a theorem we used to begin our interpolation:

The constants in the summation

$$S_n(x) = \{a_0/2 + a_n \cos(nx) + \sum_{k=1}^{n-1} [a_k \cos(kx) + b_k \sin(kx)]$$

that minimize the least square sum

$$E(a_0, .., a_n, b_1, ..., b_{n-1}) = \sum_{j=0}^{2m-1} [y_j - S_n(x_j)]^2$$

are

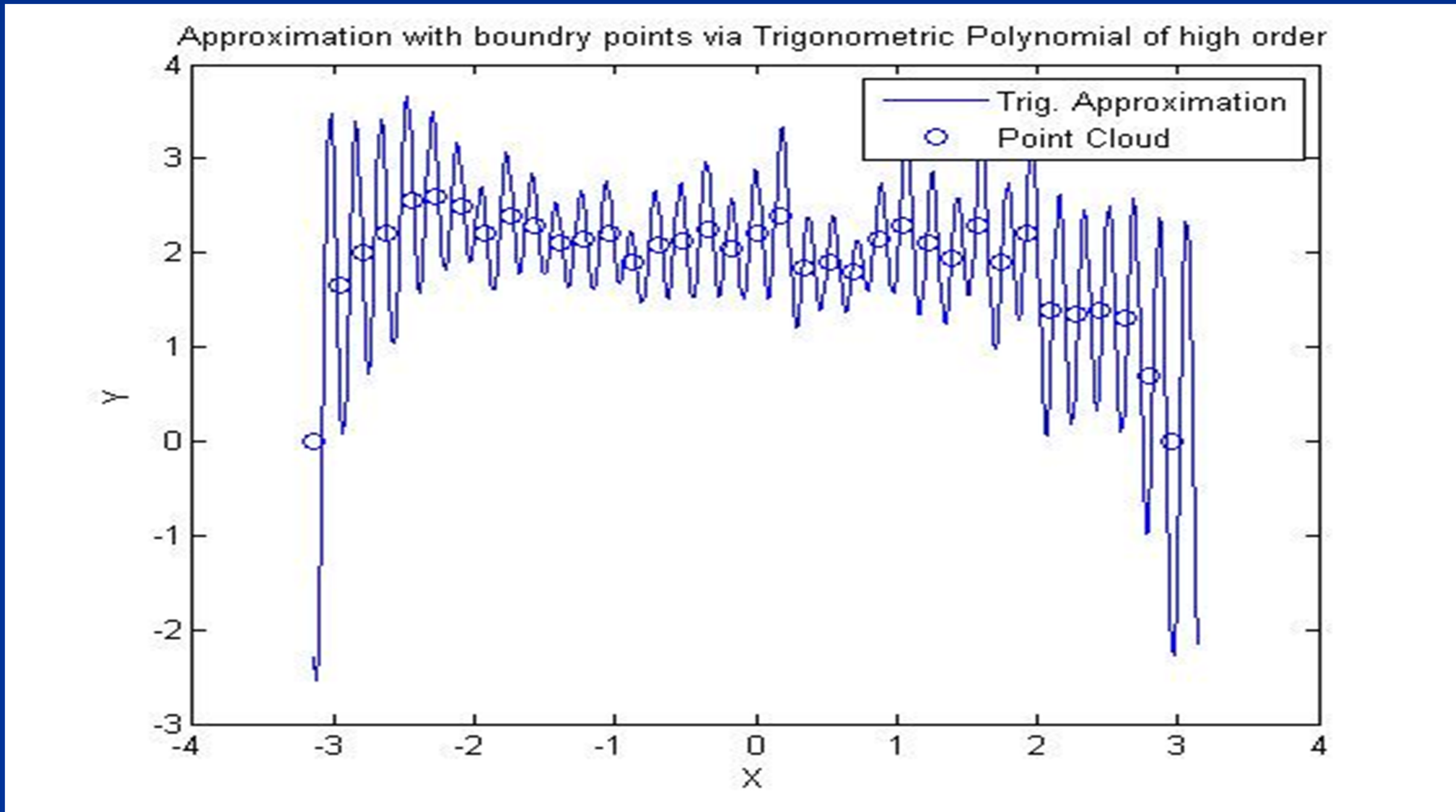
$$a_k = (1/m) \sum_{j=0}^{2m-1} [y_j \cos(kx_j)], \forall k = 0, 1, .., n$$

and

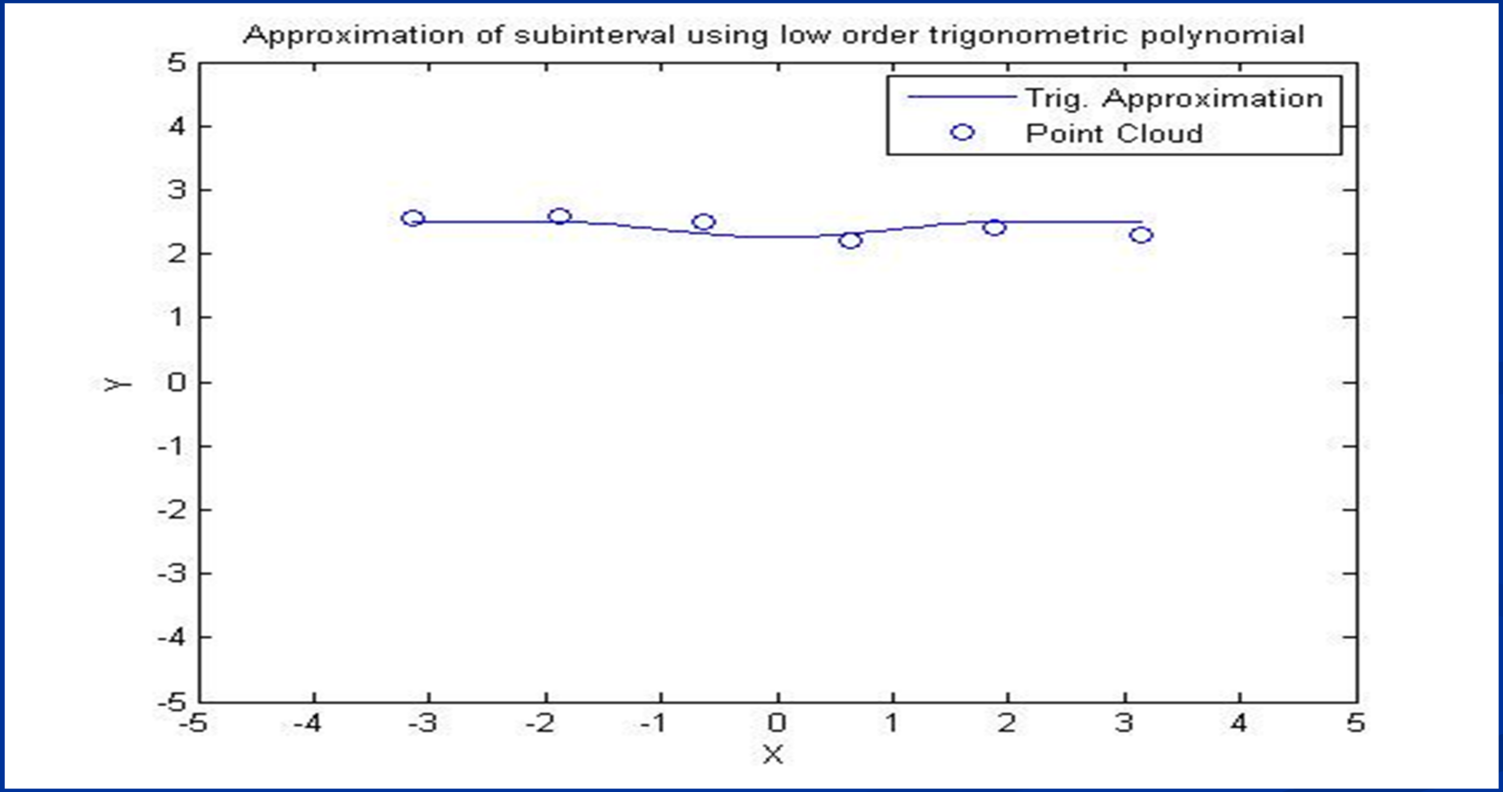
$$b_k = (1/m) \sum_{j=0}^{2m-1} [y_j \sin(kx_j)], \forall k = 0, 1, .., n-1$$

Interpolation

- Applying this idea to our point cloud, we first obtained an extremely high degree polynomial of degree 36.



- We decided to partition our domain, and interpolate the points in each sub-domain.



Future Work

- Assemble the polynomials obtained in the previous slide into piecewise continuous functions that will be used to create the mesh which best resembles the top surface of placenta.
- Do the same for the bottom surface
- Patch the top and bottom surfaces together

References

- Numerical Analysis, Richard L. Burden
- Placenta data, Dr. Chang
- http://www.cs.uregina.ca/Links/class-info/425/Lab3/