Group #: _____ Members: _____ Rating: _____

We now want to look at the solutions of the matrix equation $Ax = b$ by examining the properties of the transformation A. Particularly, I want you to look for the connection between the one-to-oneness and ontoness of the matrix transformation A with the uniqueness and existence questions.

1. (Definitions) Fill in the blanks.
   (a) (1 point) A mapping $T : \mathbb{R}^n \to \mathbb{R}^m$ is onto if _____. The standard matrix A that is associated with the linear mapping T is onto if _____.
   (b) (1 point) A mapping $T : \mathbb{R}^n \to \mathbb{R}^m$ is one-to-one if _____. The standard matrix A that is associated with the linear mapping T is one-to-one if _____.
2. (2 points) Let $T : \mathbb{R}^4 \to \mathbb{R}^3$ be the linear transformation whose standard matrix is $A = \begin{bmatrix} 1 & -4 & 8 & 1 \\ 0 & 2 & -1 & 3 \\ 0 & 0 & 0 & 5 \end{bmatrix}$.
   (a) Does T map $\mathbb{R}^4$ onto $\mathbb{R}^3$? Justify your answer (Hint: Answering the ontoness question is equivalent to answering the _____ question in the matrix equation. Hence, methods for showing the _____ question can be used here to answer the ontoness question.)
   (b) Is T one-to-one? Justify your answer (Hint: Answering the one-to-oneness question is equivalent to answering the _____ question in the matrix equation.)
3. (4 points) Find four equivalent statements (i.e., statements that are true under the if and only if condition) to each of the following statements. Make appropriate use of the associated matrix equation $Ax = b$.
   (a) A linear transformation $T : \mathbb{R}^n \to \mathbb{R}^m$ is one-to-one.
   (b) A linear transformation $T : \mathbb{R}^n \to \mathbb{R}^m$ is onto.
4. (2 points) If a linear transformation $T : \mathbb{R}^n \to \mathbb{R}^m$ maps $\mathbb{R}^n$ *onto* $\mathbb{R}^m$, give a relation between m and n. If T is one-to-one, what can you say about m and n? Illustrate your reasonings with concrete examples. If T is one-to-one AND onto, what can you say about m and n?