Group #: ________ Members: ______________________________ Rating: __________

1. (4 points) Let
$$A = \begin{bmatrix} 1 & -3 \\ 3 & 4 \\ -1 & 7 \end{bmatrix},\ u = \begin{bmatrix} 2 \\ -1 \end{bmatrix},\ b = \begin{bmatrix} 3 \\ 2 \\ -5 \end{bmatrix},\ c = \begin{bmatrix} 3 \\ 2 \\ 5 \end{bmatrix},$$
and define a transformation $T : \mathbb{R}^2 \to \mathbb{R}^3$ by $T(x) = Ax$ so that
$$T(x) = Ax = \begin{bmatrix} 1 & -3 \\ 3 & 4 \\ -1 & 7 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} x_1 - 3x_2 \\ 3x_1 + 4x_2 \\ -x_1 + 7x_2 \end{bmatrix}.$$
   (a) Find the image of u under T.
   (b) Does b have a pre-image? That is, does there exist an $x \in \mathbb{R}^2$ whose image under T is b? Why or why not? If so, find one such x. (this is an existence question!)
   (c) Is there more than one x whose image under T is b? Why or why not? (this is an uniqueness question!)
   (d) Is c in the range of T? Why or why not?

2. (2 points) Let $A = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$. Give a geometric interpretation of the mapping $x \mapsto Ax$ and be sure to show your reasonings.

3. (2 points) Let $\mathbf{e}_1 = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$, $\mathbf{e}_2 = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$, $\mathbf{y}_1 = \begin{bmatrix} 2 \\ 5 \end{bmatrix}$, and $\mathbf{y}_2 = \begin{bmatrix} -1 \\ 6 \end{bmatrix}$. Furthermore, let $T : \mathbb{R}^2 \to \mathbb{R}^2$ be a linear transformation that maps $\mathbf{e}_1$ into $\mathbf{y}_1$, and maps $\mathbf{e}_2$ into $\mathbf{y}_2$. Find the images of $\begin{bmatrix} 5 \\ -3 \end{bmatrix}$ and $\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$.

4. (2 points) Define $f : \mathbb{R} \to \mathbb{R}$ by $f(x) = mx + b$.
   (a) Show that f is a linear transformation when $b = 0$.
   (b) Is f a linear transformation in general? Why or why not? Justify your answer.
   (c) Why is f typically called a linear function?