1. (4 points) Let

\[ A = \begin{bmatrix} 1 & -3 \\ 3 & 4 \\ -1 & 7 \end{bmatrix}, \quad u = \begin{bmatrix} 2 \\ -1 \end{bmatrix}, \quad b = \begin{bmatrix} 3 \\ 2 \\ -5 \end{bmatrix}, \quad c = \begin{bmatrix} 3 \\ 2 \end{bmatrix}. \]

and define a transformation \( T : \mathbb{R}^2 \to \mathbb{R}^3 \) by \( T(x) = Ax \) so that

\[ T(x) = Ax = \begin{bmatrix} 1 & -3 \\ 3 & 4 \\ -1 & 7 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} x_1 - 3x_2 \\ 3x_1 + 4x_2 \\ -x_1 + 7x_2 \end{bmatrix}. \]

(a) Find the image of \( u \) under \( T \).

(b) Does \( b \) have a pre-image? That is, does there exist an \( x \in \mathbb{R}^2 \) whose image under \( T \) is \( b \)? Why or why not? If so, find one such \( x \). (this is an existence question!)

(c) Is there more than one \( x \) whose image under \( T \) is \( b \)? Why or why not? (this is an uniqueness question!)

(d) Is \( c \) in the range of \( T \)? Why or why not?

2. (2 points) Let \( A = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix} \). Give a geometric interpretation of the mapping \( x \mapsto Ax \) and be sure to show your reasonings.

3. (2 points) Let \( e_1 = \begin{bmatrix} 1 \\ 0 \end{bmatrix}, \quad e_2 = \begin{bmatrix} 0 \\ 1 \end{bmatrix}, \quad y_1 = \begin{bmatrix} 2 \\ 5 \end{bmatrix}, \quad \text{and} \quad y_2 = \begin{bmatrix} -1 \\ 6 \end{bmatrix}. \) Furthermore, let \( T : \mathbb{R}^2 \to \mathbb{R}^2 \) be a linear transformation that maps \( e_1 \) into \( y_1 \), and maps \( e_2 \) into \( y_2 \). Find the images of \( \begin{bmatrix} 5 \\ -3 \end{bmatrix} \)

and \( \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} \).

4. (2 points) Define \( f : \mathbb{R} \to \mathbb{R} \) by \( f(x) = mx + b \).

(a) Show that \( f \) is a linear transformation when \( b = 0 \).

(b) Is \( f \) a linear transformation in general? Why or why not? Justify your answer.

(c) Why is \( f \) typically called a linear function?