Group #: ________ Members: ______________________________ Rating: ___________

1. (2 points) Define linear independence of a set. Use your own words and define your own notations.

2. (2 points) Let

$$v_1 = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}, \quad v_2 = \begin{bmatrix} 4 \\ 5 \\ 6 \end{bmatrix}, \quad v_3 = \begin{bmatrix} 2 \\ 1 \\ 0 \end{bmatrix}.$$

Determine if $\{v_1, v_2, v_3\}$ is a linearly independent set. Justify your answer.

3. The following results help us characterize sets that are always linearly independent and sets that are always linearly dependent. Prove the following theorems.

   (a) (2 points) An indexed set $S = \{v_1, v_2, \ldots, v_p\}$ of two or more vectors is linearly dependent if and only if at least one of the vectors in S is a linear combination of the others. Give an example to illustrate the result.

   (b) (2 points) If a set contains more vectors than there are entries in each vector, then the set is linearly dependent. That is, any set $\{v_1, \ldots, v_p\}$ in $\mathbb{R}^n$ is linearly dependent if $p > n$. Give an example to illustrate the result.

   (c) (1 point) If a set $S = \{v_1, \ldots, v_p\}$ in $\mathbb{R}^n$ contains the zero vector, then the set is linearly dependent.

   (1 point) So, what kind of set would **always** be linearly independent? What kind of set would **always** be linearly dependent? Write your answer in complete sentences. Feel free to include examples in your statements.