Group #: ________ Members: ______________________________ Rating: __________

Let's get into a good habit of justifying our work. To **formally** justify a **true** *if-then* statement, you need to cite appropriate results (typically in the form of theorems) and show that the antecedent (what follows the "if") in the True/Flase statement is satisfied. This may be hard to do in some situations. On the other hand, to justify a **false** *if-then* statement, you just need to come up with **one** counterexample (i.e., example that shows the statement is not true). Note, you **can not** justify a true statement by simply showing that a few examples work (Getting heads on ten consecutive coin tosses does not mean that you won't get tails on the 11th toss.).

1. (3 points) Let A be an $m \times n$ matrix and $\mathbf{b} \in \mathbb{R}^m$. What can you say about the consistency of the system $A\mathbf{x} = \mathbf{b}$ if

   (a) A has a pivot position in every row.

   (b) the columns of A span $\mathbb{R}^m$.

   Base on your results here, construct a matrix A of any size such that the system $A\mathbf{x} = \mathbf{b}$ is consistent for every $\mathbf{b}$. Be sure to justify your reasonings.

2. (2 points) Let $\mathbf{u} = \begin{bmatrix} 2 \\ -3 \\ 2 \end{bmatrix}$ and $A = \begin{bmatrix} 5 & 8 & 7 \\ 0 & 1 & -1 \\ 1 & 3 & 0 \end{bmatrix}$. Is $\mathbf{u}$ in the subset of $\mathbb{R}^3$ spanned by the columns of A? Why or why not?

3. (3 points) Describe the solution set (geometrically and in vector form) of

   (a) the homogeneous system $10x_1 - 3x_2 - 2x_3 = 0$.

   (b) the non-homogeneous system $10x_1 - 3x_2 - 2x_3 = 5$.

   Summarize your findings.

4. (2 points) Mark each of the following (definition) statements True or False. Whenever possible, justify your answer. That is, point out what goes wrong in the false statements.

   (a) The effect of adding $\mathbf{p}$ to a vector is to move the vector in a direction parallel to $\mathbf{p}$.

   (b) The equation $A\mathbf{x} = \mathbf{b}$ is homogeneous if the zero vector is a solution.

   (c) The equation $\mathbf{x} = \mathbf{p} + t\mathbf{v}$ describes a line through $\mathbf{v}$ parallel to $\mathbf{p}$.

   (d) The solution set of $A\mathbf{x} = \mathbf{b}$ is the set of all vectors of the form $\mathbf{w} = \mathbf{p} + \mathbf{v}_h$, where $\mathbf{v}_h$ is any solution of the equation $A\mathbf{x} = \mathbf{0}$.