

Group #: \_\_\_\_\_ Members: \_\_\_\_\_ Rating: \_\_\_\_\_

- (1 point) Give an example of an augmented matrix of size  $4 \times 7$  that is in the row echelon form but not in the reduced row echelon form. Justify your answer.
- (4 points) Given a linear system of equations

$$\begin{array}{rcl} & x_3 - 2x_4 & = -3 \\ x_1 - 7x_2 & + 6x_4 & = 5 \\ -x_1 + 7x_2 - 4x_3 + 2x_4 & & = 7 \end{array}$$

- Row reduce its augmented matrix into the reduced echelon form. Circle the pivot positions in the final matrix and list the pivot columns. Be sure to mark your steps clearly.
  - Find the solution set of the system in **parametric form** and **vector equation form**.
- (3 points) (Definitions) Complete the following sentences by using some or all of the keywords: *pivot column, basic variables, free variables, augmented matrix*.
    - (existence) A linear system is consistent if and only if \_\_\_\_\_.
    - (uniqueness) A linear system is consistent with a unique solution when \_\_\_\_\_.
    - A linear system is consistent with infinitely many solutions when \_\_\_\_\_.
  - (1 point) Use the results from Question 3 to support your answer to the true/false question: If one row in an echelon form of an augmented matrix is  $[0 \ 0 \ 0 \ 5 \ 0]$ , then the associated linear system is inconsistent.
  - (1 point) A system of linear equations with more equations than unknowns is sometimes called an *overdetermined system*. Can such a system be consistent? Why or why not? Explain. Illustrate your answer with a specific system of equations.