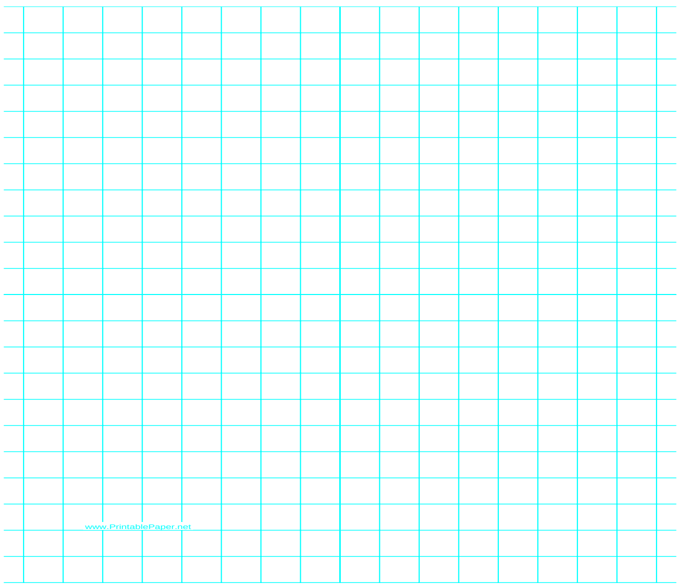
Group #: ________ Members: ______________________________ Rating: ___________

1. (4 points) Let $\mathbf{u} = \begin{bmatrix} -1 \\ 1 \end{bmatrix}$ and $\mathbf{v} = \begin{bmatrix} 2 \\ 1 \end{bmatrix}$. Answer the following questions.

   (a) (1 point) display the vectors $\mathbf{w} = \frac{5}{2}\mathbf{u} - \frac{1}{2}\mathbf{v}$ and $\mathbf{z} = \sqrt{3}\mathbf{u} + 3\mathbf{v}$ on the graph paper.




   (b) (1 point) What is the set of *all* linear combination of $\mathbf{u}$ and $\mathbf{v}$? That is, the span$\{\mathbf{u}, \mathbf{v}\}$?

   (c) (2 points) Can every point in $\mathbb{R}^2$ be written as a linear combination of $\mathbf{u}$ and $\mathbf{v}$? Justify your answer.

2. (4 points) Answer the following questions concerning span.

   (a) (1 point) Define *span* in your own words and give an example to illustrate it.

   (b) (1 point) If $a_1 = \begin{bmatrix} 1 \\ -2 \\ 3 \end{bmatrix}$, $a_2 = \begin{bmatrix} 5 \\ -13 \\ -3 \end{bmatrix}$, and $b = \begin{bmatrix} -3 \\ 8 \\ 1 \end{bmatrix}$, what is the geometric meaning of span$\{a_1, a_2\}$?

   (c) (1 point) Is $b \in \text{span}\{a_1, a_2\}$? Justify your answer.

   (d) (1 point) Use complete sentences to give a recipe for determining whether a given vector is in the span of a set of vectors.

3. (2 points) Let $A = \begin{bmatrix} 2 & 0 & 6 \\ -1 & 8 & 5 \\ 1 & -2 & 1 \end{bmatrix}$, let $\mathbf{b} = \begin{bmatrix} 10 \\ 3 \\ 7 \end{bmatrix}$. Denote the columns of $A$ by $\mathbf{a}_1, \mathbf{a}_2, \mathbf{a}_3$, let $W = \text{span}\{\mathbf{a}_1, \mathbf{a}_2, \mathbf{a}_3\}$, and $S = \{\mathbf{a}_1, \mathbf{a}_2, \mathbf{a}_3\}$.

   (a) How many vectors are in $S$? Is $\mathbf{b} \in S$? Why or why not?

   (b) How many vectors are in $W$? Is $\mathbf{b} \in W$? Why or why not?