Group #: ________ Members: ______________________________ Rating: __________

1. (Definitions) Fill in the blank with your own words. (There are often times many ways to give a definition. Copying definitions word-by-word from a textbook does not always help one understand the underlying concepts. You know that you have truly mastered the concepts when you can define them using your own words.) Be sure to carefully define notations that you use and introduce.

   (a) (1 point) A system of linear equations is consistent if ____________________.

2. (2 points) Solve the following system of equations in matrix notations. Give a geometric meaning of your solution set in words with as much detail as possible? (So detailed that someone can identify this set in space by following your descriptions.)

$$\begin{aligned} x_1 + x_2 + x_3 &= 2 \\ 2x_1 + 3x_2 + x_3 &= 3 \\ x_1 - x_2 - 2x_3 &= -6 \end{aligned}$$

3. (3 points) Given an augmented system

$$\begin{bmatrix} 1 & -2 & 1 & 0 \\ 0 & 2 & -8 & 8 \\ -4 & 5 & 9 & -9 \end{bmatrix}.$$

   Answer the following questions with as few steps as possible.

   (a) Is the system consistent? Why or why not? (This is a **existence** question)

   (b) If it is consistent, is the solution unique? Why or why not? (This is a **uniqueness** question)

4. (2 points) Find an equation involving g, h, and k that makes the following augmented matrix correspond to a consistent system.

$$\begin{bmatrix} 0 & 3 & -5 & h \\ -2 & 5 & -9 & k \\ 1 & -4 & 7 & g \end{bmatrix}$$

5. (2 points) Solve the following three systems of linear equations with as little writing as possible, all of which have the same matrix of coefficients.

$$\begin{aligned} x_1 + x_2 + x_3 &= b_1 \\ 2x_1 + 3x_2 + x_3 &= b_2 \\ x_1 - x_2 - 2x_3 &= b_3 \end{aligned} \quad \text{for} \quad \begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix} = \begin{bmatrix} 8 \\ 11 \\ -11 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 2 \end{bmatrix}, \begin{bmatrix} 3 \\ 3 \\ -4 \end{bmatrix}.$$