Group #: ________ Members: ______________________________ Rating: ___________

1. (Definitions) Fill in the blanks.
   (a) (1 point) An $n \times n$ matrix A is an orthogonal matrix if ____________________.
   (b) (1 point) A set of vectors $X = \{x_1, x_2, \ldots, x_n\}$ is an orthonormal basis for a vector space W if ____________________.

2. (5 points) Given

$$\mathbf{y} = \begin{bmatrix} 1 \\ 2 \\ 3 \\ 4 \end{bmatrix}, \quad \mathbf{u}_1 = \begin{bmatrix} 3 \\ 1 \\ -1 \\ 3 \end{bmatrix}, \quad \mathbf{u}_2 = \begin{bmatrix} -5 \\ 1 \\ 5 \\ -7 \end{bmatrix}, \quad \text{and} \quad \mathbf{u}_3 = \begin{bmatrix} 1 \\ 1 \\ -2 \\ 8 \end{bmatrix}.$$

   Let $W = \text{span}\{\mathbf{u}_1, \mathbf{u}_2, \mathbf{u}_3\}$ (space), $S = \{\mathbf{u}_1, \mathbf{u}_2, \mathbf{u}_3\}$ (set), and $U = [\mathbf{u}_1\ \mathbf{u}_2\ \mathbf{u}_3]$ (matrix).
   (a) (2 points) Verify that S is not an orthogonal set, then use the Gram-Schmidt Process to produce an **orthonormal** basis for W.
   (b) (3 points) Verify that the system $Ux = \mathbf{y}$ is inconsistent, then find its least-squares solutions two ways: (1) via the normal equation; and (2) via the QR factorization.

3. (3 points) (This is found in §6.6) Suppose that experimental data produce points $(x_1, y_1), \ldots, (x_n, y_n)$ that, when graphed, seem to lie close to a line. We want to determine the parameters β_0 and β_1 that make the line as *close* to the points as possible. This line is called the **least-squares line**, since the line is constructed by minimizing the sum of the squares of the residual. See Figure 1 for an illustration. Under this assumption, find the equation $y = \beta_0 + \beta_1 x$ of the least-squares line that best fits the data points $(1, 0)$, $(2, 1)$, $(4, 2)$, $(5, 3)$. (Make sure you graph the points along with the least-squares line to see if your solution is reasonable.)

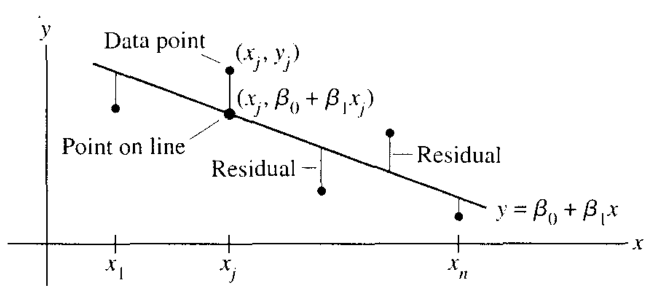

Figure 1: Fitting a least-squares line to observed data $(x_1, y_1), \ldots, (x_n, y_n)$.