Group #: ________ Members: ______________________________ Rating: __________

1. This question concerns orthogonal complement.

   (a) (2 points) Fill in the blank using the **definition**. This definition needs to be useful/functional for the subsequent problems. Let W be a subspace of $\mathbb{R}^n$, then the **orthogonal complement** of W is ______________.

   (b) (3 points) Let $W = \text{span}\{v_1, \ldots, v_p\}$. Show that if x is orthogonal to each v_j, for $1 \leq j \leq p$, then x is orthogonal to every vector in W.

   (c) (4 points extra credit) Let A be an $m \times n$ matrix. Recall that the four fundamental subspaces completely divide $\mathbb{R}^n$ and $\mathbb{R}^m$ (Row A and Nul A divide $\mathbb{R}^n$ while Col A and Nul A^T divide $\mathbb{R}^m$). Here, you will show that they do so orthogonally. Prove (i) the orthogonal complement of the row space of A is the null space of A (i.e., (Row $A)^\perp$ = Nul A or Row A $\perp$ Nul A); (ii) (as a corollary) the orthogonal complement of the column space of A is the null space of A^T (i.e., (Col $A)^\perp$ = Nul A^T or Col A $\perp$ Nul A^T). (Hint: To show two sets are equal, you need to show that everything found in set one is found in set two and vice versa.)

   (d) (3 points) Let $u_1 = \begin{bmatrix} 1 \\ 2 \\ 1 \\ 1 \end{bmatrix}$, $u_2 = \begin{bmatrix} 2 \\ 1 \\ 2 \\ 1 \end{bmatrix}$, and $W = \text{span } \{u_1, u_2\}$. Use facts in part (c) to find a basis for the orthogonal complement of W in $\mathbb{R}^4$.

2. (2 points) Verify the parallelogram law for vectors u and v in $\mathbb{R}^n$:

$$||u + v||^2 + ||u - v||^2 = 2||u||^2 + 2||v||^2$$